J&R 3.1) a) Show that $\frac{\partial AP_i}{\partial x_i} \cdot \frac{x_i}{AP_i} = \mu_i - 1$

where $\mu_i = \frac{f_i x_i}{f} = \frac{MP_i}{AP_i}$ where $MP_i = f_i$

$AP_i = \frac{f}{x_i}$

We have

$$\frac{\partial AP_i}{\partial x_i} = \frac{f_i x_i - f}{x_i^2} = \frac{1}{x_i}(MP_i - AP_i)$$

$$\Rightarrow \frac{\partial AP_i}{\partial x_i} \cdot \frac{x_i}{AP_i} = \frac{1}{x_i}(MP_i - AP_i) \cdot \frac{x_i}{AP_i}$$

$$= \frac{MP_i}{AP_i} - 1 = \mu_i - 1 \quad \underline{QED}$$

b) Since $\frac{\partial AP_i}{\partial x_i} = \frac{1}{x_i}(MP_i - AP_i)$,

we have the desired result.

J. & R. 5.2)

$$\frac{\partial}{\partial x_1} AP_1 = \frac{\partial}{\partial x_1} \frac{f(\vec{x})}{x_1} = \frac{f_1 x_1 - f(\vec{x})}{x_1^2} = \frac{1}{x_1}\left(f_1 - \frac{f}{x_1}\right)$$

If $\frac{\partial}{\partial x_1} AP_1 > 0$ then $x_1 f_1 > f(\vec{x})$. [*]

By CRS, we have $f(tx_1, tx_2) = ty$

$\Rightarrow f_1 x_1 + f_2 x_2 = y = f(\vec{x})$.

$\Rightarrow f_2 x_2 = f(\vec{x}) - f_1 x_1$

$\Rightarrow f_2 x_2 < 0$ from [*].

$\Rightarrow f_2 < 0$ since $x_2 > 0$. QED

J&R 3.5) By homo. of deg. 1, we have

$$f(t\vec{x}) = ty, \quad \forall t.$$

$$\Rightarrow \frac{\partial}{\partial t} f(t\vec{x}) = \sum_{i=1}^{m} f_i(t\vec{x})\, x_i = y = f(\vec{x})$$

Let $t=1$, and we have

$$\sum_{i=1}^{m} MP_i(\vec{x})\, x_i = f(\vec{x}) \quad \text{QED}.$$

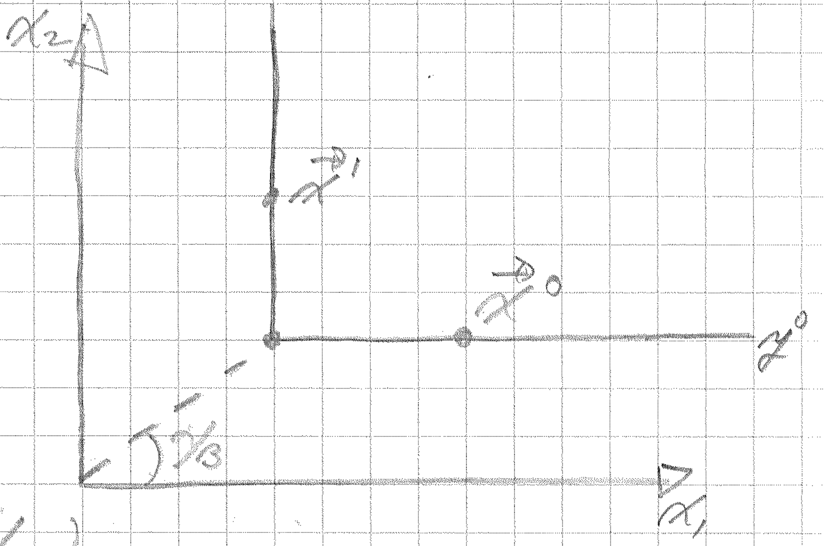
J.&R. #3.10)

$y = \min\{\alpha x_1, \beta x_2\}$, $\alpha, \beta > 0$.

$\alpha x_1 = \beta x_2 \Rightarrow y = \alpha x_1 = \beta x_2$

$\Rightarrow x_2 = \frac{\alpha}{\beta} x_1$

$\alpha x_1 \geq \beta x_2 \Rightarrow y = \beta x_2$



$$\sigma_{12} = \frac{d \ln(x_2/x_1)}{d \ln(f_1/f_2)}$$

At $\vec{x}^0$: $f_1 = 0$ and $f_2 = \beta$.

J. & R. #3.11)

$$y = A x_1^{\alpha} x_2^{\beta}, \quad A, \alpha, \beta > 0.$$

(Follow procedure in example 3.1)

$$d \ln\left(\frac{x_2}{x_1}\right) = d \ln x_2 - d \ln x_1 = \frac{1}{x_2} dx_2 - \frac{1}{x_1} dx_1$$

$$\ln\left(\frac{f_1}{f_2}\right) = \ln\left(\frac{A \alpha x_1^{\alpha-1} x_2^{\beta}}{A \beta x_1^{\alpha} x_2^{\beta-1}}\right) = \ln\left(\frac{\alpha}{\beta} \frac{x_2}{x_1}\right)$$

$$= \ln \frac{\alpha}{\beta} + \ln x_2 - \ln x_1$$

$$\Rightarrow d \ln\left(\frac{f_1}{f_2}\right) = \frac{1}{x_2} dx_2 - \frac{1}{x_1} dx_1$$

$$\Rightarrow \sigma = \frac{d \ln(x_2/x_1)}{d \ln(f_1/f_2)} = 1$$

J.R. 3.21)

$c(w, y) = A w_1^{\alpha} w_2^{\beta} y.$

In order to be increasing in $\vec{w}$, we need:

$$\frac{\partial c}{\partial w_1} = \alpha A w_1^{\alpha-1} w_2^{\beta} y \geq 0 \Rightarrow \begin{matrix} \alpha \geq 0 \\ A > 0 \end{matrix}$$

And similarly, $\beta \geq 0$.

In order to be homo. of deg 1 in $\vec{w}$, we need:

$$A\, t^{\alpha} w_1^{\alpha} t^{\beta} w_2^{\beta} y = t A w_1^{\alpha} w_2^{\beta} y$$

$$\Rightarrow t^{\alpha} t^{\beta} = t$$

$$\Rightarrow \alpha + \beta = 1$$

In order to be concave, its Hessian must be negative semi-definite. This implies:

$$\frac{\partial^2 c}{\partial w_1^2} \leq 0 \Rightarrow \alpha(\alpha-1) A w_1 w_2^{\beta} y \leq 0.$$

$\Rightarrow \alpha \leq 1$: This is already respected by $\beta, \alpha \geq 0$ and $\alpha + \beta = 1$.

In fact, it can be verified that the Hessian is neg. definite by the sufficient conditions with respect to det i-th principal minors:

$$(-1)^i D_i(x) > 0, \quad i = 1, 2.$$