J. & R. 1.56)

a) $v(\vec{p}, y) = f(y)\, p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3}$.

Given the properties in THEOREM 1.6, we must have:

① $f(y)$ continuous in y. ✓

② Homo. of deg. 0 in $(\vec{p}, y)$ implies

$$f(ty)(tp_1)^{\alpha_1}(tp_2)^{\alpha_2}(tp_3)^{\alpha_3} = f(y)\, p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3}$$

$$\Rightarrow f(ty)\, t^{\alpha_1+\alpha_2+\alpha_3} = f(y)$$

$$\Rightarrow f(ty) = t^{-(\alpha_1+\alpha_2+\alpha_3)} f(y)$$

$\Rightarrow f(y)$ is homo. of deg. $-(\alpha_1+\alpha_2+\alpha_3)$. ✓

③ v strictly increasing in y implies $f'(y) > 0$. ✓

④ v decreasing in $\vec{p}$ implies:

$\alpha_1, \alpha_2, \alpha_3 < 0$. ✓

⑤ Quasiconvex in $(\vec{p}, y)$ implies:

$v(\vec{p}, y)$ is quasi-convex iff $-v(\vec{p}, y)$ is quasi-concave.

A necessary condition for $-v(\vec{p}, y)$ to be quasi-concave is:

1.56) a) cont'd

$$(-1)^r D_r(\vec{p}, y) \geq 0$$

(Unfortunately, this is not given in Jehle & Reny.)

where
$$D_r = \begin{vmatrix} 0 & \frac{\partial \bar{v}}{\partial p_1} & \frac{\partial \bar{v}}{\partial p_2} & \frac{\partial \bar{v}}{\partial p_3} & \frac{\partial \bar{v}}{\partial y} \\ \frac{\partial \bar{v}}{\partial p_1} & \frac{\partial^2 \bar{v}}{\partial p_1^2} & \frac{\partial^2 \bar{v}}{\partial p_1 \partial p_2} & \cdots & \frac{\partial^2 \bar{v}}{\partial p_1 \partial y} \\ \frac{\partial \bar{v}}{\partial p_2} & \vdots & \ddots & & \\ \frac{\partial \bar{v}}{\partial p_3} & \vdots & & \ddots & \\ \frac{\partial \bar{v}}{\partial y} & & & & \frac{\partial^2 \bar{v}}{\partial y^2} \end{vmatrix}$$

$\bar{v} = -v$.

$$(-1)^1 D_1 = \left(\frac{\partial \bar{v}}{\partial p_1}\right)^2 = \left(\frac{\lambda_1 v(p, y)}{p_1}\right)^2 \geq 0$$

(not useful b/c always true.)

$$(-1)^2 D_2 = \begin{vmatrix} 0 & \bar{v}_1 & \bar{v}_2 \\ \bar{v}_1 & \bar{v}_{11} & \bar{v}_{12} \\ \bar{v}_2 & \bar{v}_{21} & \bar{v}_{22} \end{vmatrix} \geq 0$$

The above inequality must hold for $-v(\vec{p}, y)$ to be quasiconcave. It is a long procedure, but that's how I would do it...

1.56 b)

$v(p_1, p_2, y) = u(p_1, p_2) + z(p_1, p_2)/y$

① Continuous. ✓

② homo. of deg 0 in $(\vec{p}, y)$:

$u(tp_1, tp_2) + \frac{z(tp_1, tp_2)}{ty} = u(p_1, p_2) + \frac{z(p_1, p_2)}{y}$

$\Rightarrow u(p_1, p_2)$ is homo. of deg. 0 in (p_1, p_2).

$z(p_1, p_2)$ is homo. of deg. 1 in (p_1, p_2).

③ v strictly increasing in y implies:

$z(p_1, p_2) < 0$.

④ v decreasing in $\vec{p}$ implies:

$\frac{\partial u}{\partial p_i} < 0$, $i = 1, 2$. (Suppose $y \to \infty$.)

$\frac{\partial z}{\partial p_i} < 0$, $i = 1, 2$. (Suppose $y \to 0$.)

⑤ Quasiconvex in $(\vec{p}, y)$:

chk $-v(\vec{p}, y)$ quasi-concave?

$-v = -u(p_1, p_2) - \frac{z(p_1, p_2)}{y}$

A necessary condition:

$$(-1)^n D_n \geq 0$$

$$D_n = \begin{vmatrix} 0 & -\left(u_1+\frac{z_1}{y}\right) & -\left(u_2+\frac{z_2}{y}\right) & \frac{z}{y^2} \\ -\left(u_1+\frac{z_1}{y}\right) & -\left(u_{11}+\frac{z_{11}}{y}\right) & -\left(u_{12}+\frac{z_{12}}{y}\right) & \frac{z_1}{y^2} \\ -\left(u_2+\frac{z_2}{y}\right) & \cdot & \ddots & \vdots \\ \frac{z}{y^2} & \cdots & \cdots & -\frac{2z}{y^3} \end{vmatrix}$$

WOW!