Solution to Assignment 6, Fall 2009
MAT2377C

8.52. 8-52 a) 95

$$\hat{p} = \frac{18}{50} = 0.36, n = 50, z_{\alpha/2} = 1.96.$$

$$0.36 \pm 1.96\sqrt{\frac{(0.36)(0.64)}{50}}$$

This gives $[0.227, 0.493]$.

b)

$$n = \left(\frac{z_{\alpha/2}}{E}\right)^2 p(1-p) = \left(\frac{1.96}{0.02}\right)^2 0.36(1-0.36) = 2212.76.$$

c)

$$n = \left(\frac{z_{\alpha/2}}{E}\right)^2 p(1-p) = \left(\frac{1.96}{0.02}\right)^2 0.5(1-0.5) = 2401.$$

8.78. $\mu = 50$ and σ unknown 8-78 a)

$$n = 16\bar{x} = 52s = 1.5$$

$$t_{observed} = \frac{52-56}{8/\sqrt{16}} = 1.$$

The P-value for $t_{observed} = 1$, degrees of freedom $= 15$, is between 0.1 and 0.25. Thus we would conclude that the results are not very unusual. b) n = 30

$$t_{observed} = \frac{52-56}{8/\sqrt{30}} = 1.37$$

The P-value for $t_{observed} = 1.37$, degrees of freedom $= 29$, is between 0.05 and 0.1. Thus we conclude that the results are somewhat unusual. c) $n = 100$ (with $n > 30$, the standard normal table can be used for this problem)

$$z_{observed} = \frac{52-56}{8/\sqrt{100}} = 2.5$$

The P-value for $z_{observed} = 2.5$, is 0.00621. Thus we conclude that the results are very unusual. d) For constant values of $\bar{x}$ and s, increasing only the sample size, we see that the standard error of $\bar{X}$ decreases and consequently a sample mean value of 52 when the true mean is 50 is more unusual for the larger sample sizes.

9.20. a)

$$\begin{aligned}\alpha &= P(X \leq 4.85 | \mu = 5) + P(X > 5.15 | \mu = 5) \\ &= P\left(Z < \frac{4.85-5}{0.25/\sqrt{8}}\right) + P\left(Z < \frac{5.5-5}{0.25/\sqrt{8}}\right)\end{aligned}$$

$$= P(Z - 1.7) + P(Z > 1.7) = P(Z - 1.7) + (1 - P(Z1.7)$$
$$= 0.04457 + (1 - 0.95543) = 0.08914$$

b)

$$\text{Power} = 1 - \beta = \beta = P(4.85 \leq \bar{X} \leq 5.15 | \mu = 5.1)$$
$$= P\left(\frac{4.85 - 5.1}{0.25/\sqrt{8}} < Z < \frac{5.15 - 5.1}{0.25/\sqrt{8}}\right)$$

$$= P(-2.83 \leq Z \leq 0.566) = P(Z \leq 0.566) - P(Z \leq -2.83) = 0.71566 - 0.00233 = 0.71333$$

Therefor

$$1 - \beta = 0.2867.$$

9.39. a.) 1) The parameter of interest is the true mean battery life in hours, μ. 2)

$$H_0 : \mu = 40$$

3)

$$H_1 : \mu > 40$$

4)

$$\alpha = 0.05$$

5)

$$z_{Observed} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

6) Reject H_0 if $z_{observed} > z_\alpha$ where $z_{0.05} = 1.65$.
7) $\bar{x} = 40.5, \sigma = 1.25$

$$z_{Observed} = \frac{40.5 - 40}{1.25/\sqrt{10}} = 1.26$$

8) Since $1.26 < 1.65$ do not reject H_0 and conclude the battery life is not significantly different greater than 40 at $\alpha = 0.05$.

b)

$$P - value = 1 - \phi(1.26) = 10.8962 = 0.1038$$

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$$\beta = \phi\left(z_{0.05} + \frac{40 - 42}{1.25/\sqrt{10}}\right) == \phi(1.655.06) = \phi(-3.41)0.000325$$

(d)

$$n = \frac{(z_\alpha + z_\beta)^2\sigma^2}{\delta^2} = \frac{(z_{0.05} + z_{0.10})^2\sigma^2}{(40 - 44)^2} = \frac{(1.65 + 1.29)^2(1.25)^2}{16} = 0.844.n = 1.$$

e) 95% Confidence Interval

$$\bar{x} + z_{0.05}\sigma/\sqrt{n} \leq \mu$$

$$40.5 + 1.65(1.25)/10 \leq \mu$$

i.e.

$$39.85 \leq \mu.$$

The lower bound of the 90% confidence interval must be greater than 40 to verify that the true mean exceeds 40 hours.

9.42. a) 1) The parameter of interest is the true mean hole diameter, . 9-42 2)

$$H_0 : \mu = 1.50$$

3)

$$H_1 : \mu \neq 1.50$$

4)

$$\alpha = 0.01$$

5)

$$z_{Observed} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

6) Reject H_0 if $z_{observed} < z_{\alpha/2}$ where $z_{0.005} = 2.58$ or $z_{observed} > z_{\alpha/2}$ and $z_{0.005} = 2.58$
7) $\bar{x} = 1.4975$, $\sigma = 0.01$.

$$z_{observed} = \frac{1.49751.50}{0.01/\sqrt{25}} = -1.25.$$

8) Since $2.58 < -1.25 < 2.58$, do not reject the null hypothesis and conclude the true mean hole diameter is not significantly different from 1.5 in. at $\alpha = 0.01$.
b)

$$p - value = 2(1 - \phi(|Z_{observed}|)) = 2(1 - \phi(1.25)) \approx 0.21$$

c)

$$\beta = \phi\left(z_{0.005} - \frac{\delta\sqrt{n}}{\sigma}\right) - \phi\left(-z_{0.005} - \frac{\delta\sqrt{n}}{\sigma}\right)$$

Since $\delta = 1.495 - 1.5$ we have

$$\beta = \phi(5.08) - \phi(-0.08) = 1 - 0.46812 = 0.53188$$

$$power = 1 - \beta = 0.46812.$$

d) Set $\beta = 1 - 0.90 = 0.10$

$$n = \frac{(z_{\alpha/2} + z_\beta)^2\sigma^2}{\delta^2} = \frac{(2.58 + 1.29)^2(0.01)^2}{(-0.005)^2} = 59.908,$$

So $n = 60$.

e) For $\alpha = 0.01, z_{\alpha/2} = z_{0.005} = 2.58$

We have

$$\bar{x} \pm z_{\alpha/2}\sigma/\sqrt{n} = 1.4975 + 2.58(0.01/\sqrt{25})$$

This gives $1.4923 \leq \mu \leq 1.5027$ The confidence interval constructed contains the value 1.5, thus the true mean hole diameter could possibly be 1.5 in. using a 99% level of confidence. Since a two-sided 99% confidence

interval is equivalent to a two-sided hypothesis test at $\alpha = 0.01$, the conclusions necessarily must be consistent.

11.41. a)

$$9.10130 \leq \beta_1 \leq 9.31543$$

b)

$$11.6219 \leq \beta_0 \leq 1.04911$$

c)

$$500.124(2.228)\sqrt{3.774609\left(\frac{1}{12} + \frac{(55-46.5)^2}{3308.9994}\right)}$$
$$= 500.124 \pm 1.4037586.$$

$$498.72024 \leq E(Y) \leq 50152776.$$

. d)

$$500.124 \pm (2.228)\sqrt{3.774609\left(1 + \frac{1}{12} + \frac{(55-46.5)^2}{3308.9994}\right)}$$
$$= 500.124 \pm 4.5505644.$$

Therefore

$$495.57344 \leq y_0 \leq 504.67456.$$

It is wider because the prediction interval includes errors for both the fitted model and for a future observation.