MAT 2377
Sample Midterm

Time: 80 minutes

Student Number:

Family Name: ____

First Name: ______

This is an open book examination.
Only non-programmable and non-graphic calculators are permitted.
Record your answer to each question in the table below.
Number of pages: **6**.
NOTE: At the end of the examination, hand in only this page. You may keep the questionnaire.

Question	Answer
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

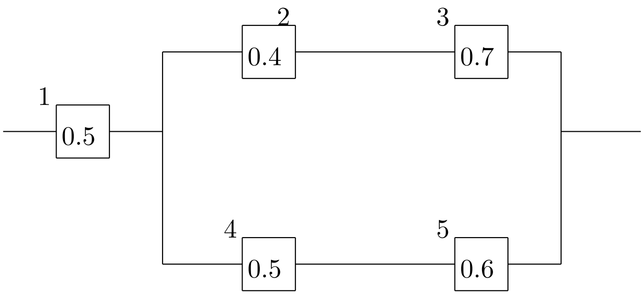
Q1. A pair of fair dice is tossed once. The probability that the sum is at most 5, is:

(a) 10/36 (b) 6/36 (c) 4/36 (d) 2/36 (e) none of the preceding

Solution to Q1:

Count number of possibilities.

Q2. Consider the following system with five components. We say that it is functional if there exists a path of functional components from left to right. The probability of each component functions is shown. Assume that the components function or fail independently. What is the probability that the system operates ?



(a) 0.841 (b) 0.165 (c) 0.248 (d) 0.5 (e) none of the preceding

Solution to Q2:

Call 'Box B' - components 2,3, 'Box C' - components 4,5

$$\begin{aligned} P(\text{Box C operates}) &= P(\text{component 4 operates and component 5 operates}) \\ &= P(\text{component 4 operates})P(\text{component 5 operates}) = 0.5 \times 0.6 = 0.3. \end{aligned}$$

$$\begin{aligned} P(\text{Box B operates}) &= P(\text{component 2 operates and component 3 operates}) \\ &= P(\text{component 2 operates})P(\text{component 3 operates}) = 0.4 \times 0.7 = 0.28. \end{aligned}$$

$$\begin{aligned} P(\text{system operates}) &= P(\text{component 1 operates})P(\text{Box B or Box C operate}) \\ &= P(\text{component 1 operates}) \times \\ &\quad \{P(\text{Box B operates}) + P(\text{Box C operates}) - P(\text{Box B operates})P(\text{Box C operates})\} \\ &= 0.5 \times (0.3 + 0.28 - 0.3 * 0.28) = 0.248 \end{aligned}$$

Q3. A manufacturer produces computers. To check customer satisfaction with a product, 100 people have been interviewed. The results are as follows:

	Satisfied	Not Satisfied
Male	19	41
Female	12	28

Given that a randomly selected customer is 'Satisfied', the probability that it is male, is:

(a) 0.19 (b) 0.3166 (c) 0.6129 (d) 0.3577 (e) none of the preceding

Solution to Q3:

Let M - 'Male', F - 'Female', S - 'Satisfied', S^c - 'Not Satisfied' To compute $P(M|S)$.

$$P(M|S) = \frac{P(M \cap S)}{P(S)} = \frac{19/100}{31/100} = 0.6129$$

Q4. An insurer has three types of auto insurance policyholders. 50% of the policyholders are at low risk (L). The probability that a low-risk policyholder will file a claim in a given year is .10. Another 30% of the policyholders are at moderate risk (M). The probability that a moderate-risk policyholder will file a claim in a given year is .20. Finally, 20% of the policyholders are high risk (H). The probability that a high-risk policyholder will file a claim in a given year is .50. A policyholder files a claim this year. The probability that he is a high-risk policyholder is

(a) 0.524 (b) 0.476 (c) 0.500 (d) 0.324 (e) none of the preceding

Solution to Q4:

Let C denotes the event that a policy holder makes a claim. To compute $P(H|C)$. Use Bayes' formula.

$$\begin{aligned} P(H|C) &= \frac{P(C|H)P(H)}{P(C)} = \frac{P(C|H)P(H)}{P(C|H)P(H) + P(C|M)P(M) + P(C|L)P(L)} \\ &= \frac{0.5 * 0.2}{0.5 * 0.2 + 0.2 * 0.3 + 0.5 * 0.1} \approx 0.476 \end{aligned}$$

Q5. From past experience it is known that 3% of accounts in a large accounting population are in error. The probability that exactly 5 accounts are audited before an account in error is found, is:

(a) 0.242 (b) 0.011 (c) 0.030 (d) 0.026 (e) none of the preceding

Solution to Q5:

$X \sim$ Geometric.

$$P(X = 5) = P(\text{1st 4 are correctly stated })P(\text{5th in error}) = 0.97^4(0.03) \approx 0.026$$

Q6. Let X be a discrete random variable with range $\{0, 1, 2\}$ and probability mass function (p.m.f.) given below.

x	0	1	2
$f(x)$	0.5	0.3	0.2

The expected value and the variance of X are, respectively:

(a) 0.7, 0.61 (b) 0.7, 1.1 (c) 0.5, 0.61 (d) 0.5, 1.1 (e) none of the preceding.

Q7. Consider a random variable X with the following probability density function:

$$f(x) = \begin{cases} 0 & \text{if } x \leq -1 \\ \frac{3}{4}(1-x^2) & \text{if } -1 < x < 1 \\ 0 & \text{if } x \geq 1 \end{cases}$$

The value of $P(X \leq 0.5)$ is

(a) 11/32 (b) 27/32 (c) 16/32

(d) 1 (e) none of the preceding

Solution to Q7:

To compute:

$$P(X \leq 0.5) = \int_{-1}^{0.5} \frac{3}{4}(1-x^2)\,dx = \frac{3}{4}x|_{-1}^{0.5} - \frac{1}{4}x^3|_{-1}^{0.5} = 27/32.$$

Q8. A receptionist receives on average 2 phone calls per minute. If the number of calls follows a Poisson process, what is the probability that the waiting time for call will be greater than 1 minute?

(a) $e^{-1/15}$ (b) $e^{-1/30}$ (c) e^{-2} (d) e^{-1} (e) none of the preceding

Solution to Q8:

We have Poisson process with $\lambda = 2$. Now, waiting time in Poisson process is exponential. Let X be an exponential random variable with the parameter $\lambda = 2$. To compute: $P(X > 1) = \exp(-2 \times 1)$.

Q9. In a group of 70 students, each student has a probability of 0.7 of passing an exam. The approximate probability that more than 50 of them will pass an exam, is:

(a) 0.983 (b) 0.017 (c) 0.652 (d) 0.348

(e) none of the preceding

Solution to Q9:

X- number of students who pass an exam. $X \sim \mathcal{B}(70, 0.7)$, $E(X) = np = 49$, $\text{Var}(X) = 14.7$. To compute $P(X > 50) = 1 - P(X \leq 50)$:

$$P(X \leq 50) = P\left(\frac{X - np}{\sqrt{np(1-p)}} < \frac{50 + 0.5 - 49}{\sqrt{14.7}}\right) \approx P(Z < 0.39) = 0.651732.$$

Q10. A company manufactures hockey pucks. It is known that their weight is normally distributed with mean 1 and the standard deviation 0.05. The pucks used by NHL must weight between 0.9 and 1.1. What is the percentage of pucks produced by the company, which could be used by NHL?

(a) 100% (b) 95.45% (c) 4.56% (d) 97.72% (e) none of the preceding

Solution to Q10:

$$P(0.9 < X < 1.1) = P(\frac{0.9-1.0}{0.05} < Z < \frac{1.1-1.0}{0.05}) = \Phi(2) - \Phi(-2) = 0.977250 - 0.022750 = 0.9545$$

This is the last question

Solutions to multiple choice questions:

$$
\begin{aligned}
\text{Q1} &\longrightarrow \text{a}\\
\text{Q2} &\longrightarrow \text{c}\\
\text{Q3} &\longrightarrow \text{c}\\
\text{Q4} &\longrightarrow \text{b}\\
\text{Q5} &\longrightarrow \text{d}\\
\text{Q6} &\longrightarrow \text{a}\\
\text{Q7} &\longrightarrow \text{b}\\
\text{Q8} &\longrightarrow \text{c}\\
\text{Q9} &\longrightarrow \text{d}\\
\text{Q10} &\longrightarrow \text{b}
\end{aligned}
$$