MAT 2377 (Fall 2016)

Final Exam Formula Sheet

- Addition Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- Conditional probability of A given B:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- Total probability rule:
  $P(A) = P(A \cap B) + P(A \cap B') = P(A|B)P(B) + P(A|B')P(B')$
- Bayes' rule

$$P(B|A) = \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|B')P(B')}$$

- Events A and B are independent if $P(A \cap B) = P(A)P(B)$
- Expected value of a discrete random variable X:

$$\mu = E(X) = \sum_x x f(x), \quad \text{where} \quad f(x) = P(X = x)$$

- Variance of a discrete random variable X:

$$\sigma^2 = \text{Var}(X) = \sum_x (x-\mu)^2 f(x) = \sum_x x^2 f(x) - \mu^2, \quad \text{where} \quad f(x) = P(X = x)$$

- Cumulative distribution function of a random variable X: $F(x) = P(X \le x)$
- If X is a binomial random variable with n trials and probability p of success, then

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}, \quad k = 0, 1, \ldots, n$$

  and its mean and variance are respectively $n\,p$ and $n\,p\,(1-p)$.
- If X is a geometric random variable with probability p of success, then

$$P(X = k) = (1-p)^{k-1}\,p, \quad k = 1, 2, 3, \ldots$$

  and its mean and variance are respectively $1/p$ and $(1-p)/p^2$.
- If X is a Poisson random variable with mean λ, then

$$P(X = k) = e^{-\lambda}\frac{\lambda^x}{x!}, \quad k = 0, 1, 2, \ldots$$

  and its mean and variance are both λ.

- If X has Poisson distribution (with $E[X] = \lambda > 5$) or a binomial distribution (with $n\,p > 5$ and $n\,(1-p) > 5$), then X has an approximate normal distribution. A continuity correction should be applied in the approximation.

- If X has an exponential distribution with mean $E[X] = 1/\lambda$. Its p.d.f. is

$$f(x) = \lambda\, e^{-\lambda\, x}, \quad \text{for } x > 0,$$

  and its variance is $1/\lambda^2$.

- Standardization: If X is a normal random variable with mean μ and variance σ^2, then

$$Z = \frac{X - \mu}{\sigma} \quad \text{has a standard normal distribution}$$

- Sample mean of the observations $x_1, \ldots, x_n$: $\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$

- Central Limit Theorem: for a random sample $X_1, \ldots, X_n$ from a population with a mean μ and a variance σ^2, then

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \text{ has approximately a standard normal distribution, when } n \text{ is large}$$

- The number of combinations, subsets of size r that can be selected from a set of n elements, is

$$C_r^n = \binom{n}{r} = \frac{n!}{(n-r)!\,r!}.$$

- The number of permutations, arrangements of size r that can be selected from a set of n elements, is

$$P_r^n = \frac{n!}{(n-r)!}.$$

- For $|x| < 1$, we have

$$\sum_{i=1}^{\infty} i x^{i-1} = (1-x)^{-2}.$$