

# Private Government and Urban Housing Market

PRELIMINARY AND INCOMPLETE – COMMENTS WELCOME

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## Abstract

Private governments are an increasingly popular innovation in the financing and provision of local public goods. Residential community associations are an example of a private government where residents of a neighbourhood give themselves the power to levy taxes and provide services to supplement those of their municipal government. This paper provides an overview of the private government phenomenon and then extends the model of Helsley and Strange (1998) by incorporating a housing market. This will allow analysis of property tax financing of government services. Equilibrium conditions are characterized. Numerical simulations demonstrate the interplay between consumers, the private government and the public government.

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# 1 Introduction

Urban structure has undergone periods of dramatic change and innovation, often pushed by unstoppable political and social forces. As the suburbanization of North America coincided with increasing standards of living, local governments were called to provide an ever-increasing range of services. But recently, whether spurred by ballooning budget deficits, inefficient bureaucracy or soaring property taxes, voters started demanding fiscal prudence. Local governments found themselves subject to large reductions in spending and tax revenues. In many localities, voters imposed constraints on increases in property taxes or local expenditure, some famous example being California's Proposition 13 in 1978 and Massachusetts' Proposition 2 $\frac{1}{2}$  in 1980<sup>1</sup>. Many public officials were forced to look for solutions to cut costs. However, the cost- and service-cutting agenda of local governments brought out new complaints: many residents still expected high quality public services and decried their deterioration. It was clear that cities were having a hard time adequately dealing with all the dissonant voices of the voters. Therefore local governments needed to look for new ways to meet residents' demands while appearing fiscally responsible, and one promising direction is the search for innovative ways to involve the private sector in the provision of public services.

An innovation arose in municipal governance that, in effect, took public authority and placed them into the hands of private individuals. Whether it was driven by consumer demand or developer supply, private government institutions began to spread. There have been various definitions of a private government, but the definition that I use in this paper is the following: A private government is any organization of private individuals, bound by geography, empowered by public authority to act as a government in matters of taxation, service provision and enforcement. Their relevance in the urban environment today is ever growing as these institutions take over more of the roles previously reserved for public authorities. The literature generally splits private governments into two types depending on who make up the members: residential community associations (RCAs), composed of homeowners, and business improvement districts (BIDs), composed of firms. Both subcategories share traits that characterize private governments and distinguish them from other forms of governance. This

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<sup>1</sup>Proposition 13 was approved by two-thirds of voters; it slashed property taxes in half and capped further tax increases to two per cent. Proposition 2 $\frac{1}{2}$  imposed an immediate property tax ceiling of 2.5 per cent of assessed value, and it also limited annual nominal increases of property tax to 2.5 per cent. Cutler, Elmendorf, and Zeckhauser (1999) provides an excellent study of Proposition 2 $\frac{1}{2}$ , its effects on municipal financing and why it was overwhelmingly supported. One reason is that voters perceived their high tax burden as a sign that the government was inefficiently run.

paper focuses on residential private governments, which are also known as common interest developments (CIDs) and homeowners associations (HOAs).

RCAAs are a rapidly growing form of residential community development, and they range widely in size from a few units to a community of thousands. Their proliferation is controversial. On one hand, they are private associations that provide desirable public services that may otherwise not be provided, but on the other hand, their abilities to impose and enforce regulations seem to be usurping public authority. Nevertheless, the demand for these communities remains staggeringly high: half of all new housing in the United States is constructed inside an RCA (Community Associations Institute 2003). Thus, it seems that residents of these communities are voluntarily subjecting themselves to another level of government. What are the implications of these institutions on the existing urban governance structure, and what will be the interaction between private and public governments? This paper addresses these issues in the context of an additional question: how does the housing market affect private governments?

This paper examines how, in the presence of a housing market and distortionary property taxes, a public municipal government interacts with a private government. I show that the presence of the property tax distorts two choices: the consumers' housing consumption decisions and the optimal provision of public services. A modified Samuelson rule characterizes equilibrium provision of public goods with elasticities playing a key role.

The paper is organized as follows. In Section 2, I present a brief survey of private government, which includes some facts and figures and a review of previous research on private government and housing markets. This is done with the goal of integrating the two types of models into one model of private government. I outline my model in Section 3. My model largely works off the model of Helsley and Strange (1998), but it also incorporates elements of multicommodity general equilibrium models of local public good provision, after Tiebout (1956). After highlighting the basic model and its implications, I give an example of equilibrium determination by performing some numerical simulations on a solved functional form in Section 4. Section 5 concludes.

## 2 A Survey of Private Governments

### 2.1 What Are Private Governments?

I begin this section by characterizing a residential private government and distinguishing this type of institution from other methods of public service provision. The clearest way to do this is to highlight five common characteristics that all private governments exhibit <sup>2</sup>:

First, membership in the private government should be *voluntary*. This appeals to the free mobility assumption that is common in multi-community local public good models stemming from Tiebout (1956). However, “voluntary” refers to the fact that if one does not want to join the private government, one can move away from its jurisdiction. If the public services provided by the private government are consumed, there is an obligation to participate in the private government <sup>3</sup>. In determining the entitlement to private government services, geography plays the foremost role. Once the boundaries of a private government are set, the services that it provides are available to all within the boundaries nonrivalrously.

The next characteristic, which is closely related, is *exclusivity*. It must be possible to exclude the consumption of the supplementary services to non-members of the private government. An extreme example is a residential gated community, which denies even the access to its streets to non-residents. By their limited geographic nature, exclusivity is a characteristic exhibited by many local public goods. Residential associations, for example, can easily make sure that their swimming pools, extra garbage pickup and security patrols are consumed only by their members.

Third, private governments provide services that are *supplementary*; that is, they are not meant to replace the traditional municipal government. Indeed, private governments initially arose because the public sector was perceived to be unresponsive to consumers’ demands for recreational and leisure facilities (Mallett 1993). Planned communities for families and retirees often advertise their extensive leisure facilities, which may include golf courses, tennis courts, swimming pools and parkland. Another popular service in residential communities is the 24-

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<sup>2</sup>These five traits were formulated by Helsley and Strange (1998).

<sup>3</sup>Reichman (1976) notes that there is a key legal issue in the regulation and operation of private governments. While municipalities are authorized to enforce payment of property taxes and obedience of by-laws, how can institutions of private individuals do the same? The most common method of enforcement, Reichman notes, is that the private government is entitled to place a lien on the offending property. However, as Mallett (1993) states, the blurred line between private and public means that private governments may lack the authority to perform these governing activities. On the other hand, we must realize that this legal situation can be subject to abuses. A primary legislative goal is thus the effective reconciliation of private institutions and public authority.

hour patrol that guards the entrances to the development, which supplement police services provided by the municipal government. Thus a member of a private government remains a resident of the municipality, and therefore continues to pay property taxes to the local government and receive its services <sup>4</sup>.

Fourth, private governments should also be *self-financing*, which adds to the autonomy that they enjoy. There are three main ways that private governments raise their revenues. They may require the payment of a fixed fee, usually called an assessment, from each of its members. In many residential associations, each homeowner pays an assessment every month for the privilege of membership in the RCA. Other methods of financing include a surcharge on the property tax and user fees for the use of certain services. The revenues that private governments collect can be substantial, with the average monthly assessment for a US household being \$180 (CAI Research Foundation 1999). As hundreds or thousands of households can make up an RCA, budgets can rival those of small municipalities.

Finally, a private government possesses is *scope for strategic interaction with the public sector*. It is this interaction that differentiates the private government from other types of private sector provision. For instance, in most models where the consumer has a choice between consuming a publicly or a privately provided public good [For example, Epple and Romano (1996); Glomm and Ravikumar (1998); Gouveia (1997)], the private provider of the service has no objective of its own. Rather, these models assume that consumers can freely purchase as much or as little of the private alternative as they desire. Indeed, in most models of local public good provision, even the traditional public government is often a passive agent. In each community, the median voter decides on the level of public good that should be provided, and the government's budget constraint and production technology imply automatically the level of taxation. Yet in the study of private governments there are some reasons why governments should be viewed as strategic players in the public good provision game.

First, the nature of the services a private government provides prevents members from consuming as much or as little of the public good as they want. Residential associations sell themselves on their security patrols and their recreational facilities, and the decision of membership in a private government implies an all-or-nothing consumption of the package

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<sup>4</sup>Note, however, that not all residential private governments coexist with public governments. This is because they may be located in unincorporated territory; in this case they usually receive core public services (water, sewers, etc.) from a higher level of government, such as a province, county or state. While these represent a minority of existing RCAs, they can be viewed as a developers' decision to withdraw from interacting with a traditional local government.

of services the government offers. While it is conceivable that certain services such as home security or advertising can be procured in variable amounts by individuals, the purpose of the private government model is to examine what motivates a group of private individuals to purchase services collectively. The second reason for considering strategic governments is to examine the widely-held belief that the existence of a private alternative threatens the traditional public provision. By modelling the private and the public governments as economic actors, we can see if this shifting of responsibility is indeed motivated and justified.

The five characteristics mentioned above are broad enough to accommodate a range of institutions but are sufficiently restrictive so as to rule out traditional models of private local public good provision, such as clubs. They also differentiate private governments from simply another layer of public government because they supplement existing local services rather than providing a new layer of services.

Legally, a residential private government involves each homeowner having a separate interest in his own unit and an undivided interest in common with other owners. This common property is managed by the RCA, of which the homeowner is a member. Residential private governments are classified into three main types, based on the legal definition of the common property owned by the homeowners association. There is also a natural distinction among the types:

**Condominiums:** Each homeowner has individual ownership of a unit and is a tenant in common ownership of the common property.

**Cooperatives:** A corporation holds title to the units and common area, and each household is granted exclusive rights to occupy a unit.

**Planned unit developments:** Each homeowner has individual ownership of a unit, and a corporation has title to the common areas.

**Source:** Community Associations Institute (2003)

## 2.2 The Growing Trend of Private Governments

Private governmental institutions are growing in relevance in urban settings, and their proliferation is changing people's view of local government; this should guide further research of this phenomenon. To motivate this, according to the 2002 US Census of Governments, there

are 87,849 units of local governments in the United States. These include county governments, general purpose local governments, special purpose districts and school districts. In comparison, the Community Associations Institute estimates that there are 231,000 residential private governments. Putting this number in perspective, there were only 500 associations in 1965 and 10,000 in 1970. Given this kind of growth and spread, it is clear that private governments' impact on the traditional urban public sector cannot be neglected. Yet not one of these private governments is counted by the Census of Governments, while they all provide, to some extent, services that are also provided by the traditional public sector.

As was noted previously, half of all new housing construction in the United States lies within a private government association, while Barton and Silverman (1994) state that this number approached 80 per cent in fast-growing suburban California counties. This phenomenon is also documented by the 2001 American Housing Survey. Tables 1 and 2 provide some summary values for this survey. Table 1 reports the percentage of owner-occupied households that reported paying a housing association (or RCA) fee in 2001, while Table 2 adds in those homeowners that pay a condominium or cooperative fee. While nine per cent of all American homeowners belong to a housing association, if only units less than four years old are considered, then 28 per cent of these households are members of a housing association. If condominiums and cooperatives are included in the sample, then 34 per cent of new housing, or 14 per cent of overall housing, belong to residential private governments.

Regional differences also underline this proliferation. Residential private governments are most common in the South and the West. As these regions are also the fastest-growing regions in the US, it is not surprising that the percentage of new housing in private governments has increased so dramatically.

There is also a major distinction between housing associations in central cities and in the suburbs. Table 4 shows the number of incorporated residential private governments for Los Angeles and San Francisco, and their surrounding suburban counties. In the central cities, it is clear that private governments tend to be condominiums and to have a small number of units. In contrast, planned developments are much more numerous in suburban counties, and there also tends to be a greater number of big developments. I offer several explanations for this. The conventional view is that the popularity of private governments is a response to suburban residents' demand for higher levels of service than that offered by a central city. On the other hand, it may be due to the fact that many suburbs are located in jurisdictions

where county provided services may not be sufficient. Finally, the housing association may have been incorporated as part of a large-scale housing development, which are more prevalent in the suburbs than in central cities. Construction of large-scale housing developments is not generally feasible in the central city, but that of compact condominium complexes is.

### 2.3 Previous Research on Private Government

An examination of economic research on private government reveals the surprising fact that there has been little theoretical research and hardly any empirical research on private governments. Perhaps this is due to the newness of the concept and the vague area between the public and private sector. Theoretical models in economics may have tended to group these governments in with more traditional models of club goods; however, as I have explained above, private governments have specific characteristics that distinguish them from other forms of public good provision. Research has instead been based largely in the political science and urban planning literature; as such, it is mostly descriptive and heavily based on case studies.

Helsley and Strange (1998) is the first paper that models private government as a distinct institution that has both public and private characteristics. Its main contribution is highlighting the strategic interaction that takes place between consumers, the private government and the traditional public government. This strategic policy making is attested in the planning literature. They consider only one urban area, where there is a local government providing a public good. The local government's objective is to maximize the aggregate welfare of the urban population. The novelty of their model is that they allow the formation of a private government, whose purpose is to maximize the welfare of its members. The private government provides a supplementary level of the public good that is exclusively available to members. Rather than modelling this in general equilibrium, Helsley and Strange think of the model as a game, in which the three players — consumers, the public government and the private government — interact strategically in determining their actions. The principal implication of the game is that the public government scales back its provision, better suiting the desires of nonmembers, in response to the existence of a private government. The existence of this “strategic downloading” demonstrates that viewing either the public or the private government as non-strategic, passive players misses a key element of the public good provision problem.

It is important to note that the interaction between public and private providers of a public good is not a new area of study. Papers in the study of local public goods, especially those

that follow in the footsteps of the multi-community model of Tiebout (1956), have examined how equilibrium outcomes can change when there is a private provider. In Epple and Romer (1991), for example, consumers must choose either a public provider of health care or opt out in favour of private provision. If the consumer chooses private provision, he or she can consume as much or as little as desired. In contrast, private government studies such as Helsley and Strange (1998), Helsley and Strange (2000) and this paper assume that the private government produces a fixed amount of the public good offered to its members. That is, the governments act as separate players in this model, maximizing some objective function through its choice of public provision. There is good reason to think that this is an appropriate way to model private government expenditures, as they often provide local services that a consumer cannot purchase in variable amounts. Only by joining a private government can consumers enjoy the supplementary services they desire.

The strategic interaction between private and public governments is addressed in further detail in a later paper by Helsley and Strange (2000). In this paper they focus on the formation of private governments due to cost differentials. Private governments are assumed to have lower costs of service provision than public governments. When this outweighs the social cost of forming the private government, it is optimal to have a private government forming. However, the public sector, seeking to protect its rent-seeking position, may have an incentive to block the formation of the private government. This may result in an inefficient outcome, where the private government ought to be formed but is not allowed to.

A key distinction in my model is the addition of a housing market. Why incorporate a housing market? One reason is that it aids in integrating this model with the Tiebout models of local public good provision. As authors such as Rose-Ackerman (1979), Epple, Filimon, and Romer (1984) and Hansen and Kessler (2001) have noted, the addition of housing complicates these models, but they act as a stabilizing force that promotes the existence of equilibrium. Another reason is that I wish to understand another motivation consumers have for joining a private government. Popular opinion, especially within RCA communities, is that private governments protect and increase property values. Reichman (1976) states that “The sole purpose of the general scheme is to preserve the community as planned and so to secure property values and desired living conditions.” Indeed, a 1999 survey of RCA homeowners echoes this sentiment (CAI Research Foundation 1999). It reports that 85 per cent of RCA members believe their property values are rising, and also that a primary reason for purchasing

a home in an RCA is that it represents a good investment. Thus, in my model I should expect that the value of the extra services provided by the private government be reflected in housing prices. However, in the presence of distortionary property taxes, this capitalization may not be perfect.

### 3 Model

My model of private government with a housing market extends the features of the private government model formulated by (Helsley and Strange 1998). After outlining the basic elements of my model, I discuss some important assumptions. I then characterize the equilibrium conditions and demonstrate the conditions under which stable equilibria arise.

#### 3.1 Players

The model consists of one urban area, whose land area is fixed. There are three main players: consumers who dwell in the city, the public government and the private government. I examine each in turn.

##### Consumers

The population of the city is fixed and normalized to 1. Each consumer has the identical utility function  $U(x, h, G)$ , where  $h$  represents housing services,  $G$  is the level of government services in the form of a local public good and  $x$  is a numeraire composite good. The value of  $h$  can vary continuously and can be thought of as the quantity or quality of housing services.  $U(x, h, G)$  is increasing, strictly quasiconcave and twice differentiable in all its elements. To address the issue of housing rents, I make the initial usual assumption that all consumers in the city are renters, and their rents accrue to absentee landlords residing outside the city. The consumers differ in their exogenous income  $y^i$ . This exogeneity and the absentee landlord assumption imply that income does not depend on housing market conditions. The distribution of incomes is denoted  $f(y)$  with support  $[\underline{y}, \bar{y}]$ , so that  $\int_{\underline{y}}^{\bar{y}} f(y^i) dy^i = 1$ . I refer to each consumer by income,  $y^i$ . In this model, consumers must do two things: first they choose whether or not to join the private government, and then they choose utility-maximizing levels of  $x$  and  $h$ . Regardless of whether a consumer chooses to join or not to join the private government, each will nevertheless receive some level of  $G$  from the public government. A consumer can only

consume a higher level of  $G$  by joining the private government.

Two assumptions about  $U(x, h, G)$  are needed for the analysis, and it is useful to mention them now.

**Assumption 1**  $U(x, h, G)$  is homothetic.

**Assumption 2**  $M(\tilde{r}, G, y) \equiv \frac{d\tilde{r}}{dG}|_{V=\bar{v}}$  is increasing in  $y$ .

Assumption 1 is made for convenience in deriving the equilibrium conditions of the model. While it is not innocuous, it has been commonly assumed in other models of local public good provision (see, for example, Hansen and Kessler (2001)) Assumption 2 is the important *single-crossing* condition (as used by Epple, Filimon, and Romer (1983) and others). This assumption states that the slope of a consumer's indifference curve in  $(G, \tilde{r})$  space increases as income increases. Here  $\tilde{r}$  represents the gross-of-tax price of housing. These assumptions are useful because they give us the desired sorting result – that there exists a marginal consumer who is indifferent between membership and nonmembership in the private government. This consumer partitions the population into a set of members (those with income higher than the marginal consumer) and a set of nonmembers (those with income lower than the marginal consumer).

## The Public Government

The public government is obliged to provide a level of the public good to all inhabitants of the city. I call  $g$  the level of the public good that is provided by the public government. The average cost of production for each unit of  $g$  is  $c$ . In order to finance provision, the public government levies an ad valorem property tax  $t$  on all housing services consumed in the city. As the public government's budget must balance, choosing  $g$  automatically determines the tax rate  $t$ . Remember that even private government members consume  $g$ ; this model rules out instances of secession. I assume that the objective of the public government is to maximize the aggregate welfare of all the residents in the city. It is also possible to assume other governmental objectives; indeed, an advantage of my general specification allows me to analyse the implications of other governmental objectives, for example, majority voting and entrepreneurial governments, but I do not consider them here.

## The Private Government

There exists a private government that determines the level of the public good that will be available only to members of the private government <sup>5</sup>. I will sometimes call this level, denoted  $\gamma$ , the "private supplement." Hence, each member of the private government will consume  $G = g + \gamma$  in total, while each nonmember will only consume  $G = g$  <sup>6</sup>. There is no cost advantage to the private government, which also produces  $\gamma$  at the constant average cost of  $c$ . Instead of financing with a property tax, however, the private government simply charges each private government member the average cost  $c$  per unit of  $\gamma$  produced. This is in line with the usual way in which homeowner associations are financed, through a fixed assessment that does not differ across households. One question that can be examined with this model, then, is how these two different methods of financing differ. In this model I assume that the objective of the private government is to maximize the aggregate welfare of its members. Again, other objective functions can be used instead.

### 3.2 Timing

The model consists of two stages:

Stage 1: I suppose the pre-existence of the three players, consumers, the public and the private governments <sup>7</sup>. The values of three variables are determined simultaneously. First, the consumers sort themselves into members or nonmembers. This amounts to determining  $y^*$ , the marginal member who is indifferent between joining and not joining the private government. This determination of  $y^*$  is done, taking  $g$  and  $\gamma$  as given. Second, the public government chooses  $g$ , taking  $y^*$  and  $\gamma$  as given. Third, the private government chooses  $\gamma$ , taking the values of  $y^*$  and  $g$  as given. Thus the equilibrium values of  $y^*$ ,  $g$  and  $\gamma$  are resolved as a Nash

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<sup>5</sup>I assume here that the private government exists for granted. Hence this is not a model of private government formation. See Helsley and Strange (2000) for such a model.

<sup>6</sup>This model assumes that the publicly- and privately-provided public goods are pure substitutes. This follows Helsley and Strange's (1998) formulation of the public good. In this sense, the term "private supplement" is apt. However, it is plausible to think of examples of publicly-provided and privately-provided local services that are not substitutes. For example, public policing and private security services may be viewed as complementary public goods. It would then be more sensible to think in terms of a general public good function  $G(g, \gamma)$  that could exploit this complementarity.

<sup>7</sup>This is in contrast to Helsley and Strange (1998), who add a stage before this one where the public government decides whether or not to allow the private government to form. For now I choose to focus only on the provision problem of the two governments as I am mainly interested in how the private government affects the housing market. In addition, in practice public governments often do not have such unilateral authority in the creation of private governments. While their formation is promulgated by law, it is often under provincial or state regulation, not local. As well, after the law is passed, creation of private governments tends to be initiated by private individuals and not the public government.

equilibrium triplet.

Stage 2: In the second stage, the consumers, having decided on membership, now maximize their utility by choosing bundles of  $x$ , the composite good, and  $h$ , housing services. Finally, the housing market clears and resolves housing prices.

### 3.3 Stage 2: Clearing of the Housing Market

I now characterize the solution of the model by beginning with the last stage. Within each stage, I will first examine the case in which there is only a public government maximizing total welfare. This *Base Case* will be useful in comparing the equilibrium conditions of this case with a private government.

#### 3.3.1 Stage 2 Without Private Government

The second stage of the model is quite simple for the base case without a private government. There is only one nonexcludable public good financed by a property tax on housing services. Housing services have the same price throughout the city, so that the problem of each consumer  $y^i$  is simply

$$\max_{x^i, h^i} U(x^i, h^i, g) \quad \text{s.t.} \quad y^i = x^i + \tilde{r}h^i \quad \text{where } g \text{ is fixed, and } \tilde{r} = (1+t)r \quad (3.1)$$

Note that in the base case,  $g$  is the level of public services, solely provided by the public government and financed by the property tax  $t$ . Only one housing price  $r$  prevails in the market. Setting up the constrained maximization problem for each household is straightforward and clearly gives rise to the usual “MRS equals price ratio condition,” namely  $\frac{U_h}{U_x} = \tilde{r}$ . Each consumer will have a housing demand function  $\hat{h}(\tilde{r}, g, y^i)$ . However, because the utility function is homothetic, the housing demand function is multiplicatively separable in prices and income. Therefore I can write consumer  $y^i$ 's housing demand in the form  $\hat{h}(\tilde{r}, g, y^i) = \phi(y^i)h(\tilde{r}, g)$ , where  $\phi' > 0$ . Note that the  $h$  function is common to everybody; only the arguments are different. I will sometimes refer to this as the unit housing function. Now the indirect utility function of each consumer is

$$V(\tilde{r}, g, y^i) = U(y^i - \tilde{r}\phi(y^i)h(\tilde{r}, g), \phi(y^i)h(\tilde{r}, g), g) \quad (3.2)$$

Finally, clear the housing market, resolving the net-of-tax housing price. In housing equilibrium, aggregate housing demand must equal aggregate supply, characterized by the following equation:

$$\int_y^{\bar{y}} \phi(y^i) h((1+t)r, g) f(y^i) dy^i = H^S(r) \quad (3.3)$$

$H^S$  represents a generic upward-sloping housing supply function that depends on net-of-tax house prices. Now I use the separability of the housing demand function to rewrite the equilibrium housing price equation:

$$h((1+t)r, g)\Phi = H^S(r) \quad \text{where} \quad \Phi \equiv \int_y^{\bar{y}} \phi(y^i) f(y^i) dy^i \quad (3.4)$$

I introduce the shorthand  $\Phi$  to represent an aggregate income measure of the population that depends only on the income distribution and its support. Equation (3.4) is enough to determine the equilibrium  $r$ , which depends on the variables  $t$  and  $g$ . These are chosen in the preceding stage. I now describe Stage 2 when there does exist a private government.

### 3.3.2 Stage 2 With Private Government

When a private government exists alongside a public government, the final stage of the model is slightly more complicated. By this time consumers have already decided whether they wish to join the private government. As I mentioned earlier, this is captured by the existence of  $y^*$ , the marginal consumer who is indifferent between membership and nonmembership. In this last stage, I assume that  $y^*$  exists, so that the set of nonmembers is  $[\underline{y}, y^*)$  and the set of members is  $[y^*, \bar{y}]$ . (For completeness, I assume that the marginal member does join the private government.) I will show later that this partitioning always looks like this; that is, consumers with  $y < y^*$  never join the private government, while those with  $y > y^*$  always join. The problem of each consumer  $y^i$  is stated as the following:

$$\max_{x^i, h^i} U(x^i, h^i, g) \quad \text{s.t.} \quad y^i = x^i + (1+t)r^N h^i \quad \text{if a nonmember} \quad (3.5)$$

$$\max_{x^i, h^i} U(x^i, h^i, g + \gamma) \quad \text{s.t.} \quad y^i = x^i + (1+t)r^M h^i + c\gamma \quad \text{if a member} \quad (3.6)$$

Recall that  $g$  is the level of public good provision by the public government, and  $\gamma$  is the level of the private government supplement. The public government's tax rate is  $t$ , while  $r^N$  is the net-of-tax price of housing services for nonmembers, and  $r^M$  is the net-of-tax price of housing services for members. I will also use the notation  $\tilde{r}^i, i = N$  or  $M$  to indicate the gross-of-tax price of housing for nonmembers and members, respectively.

Solving (3.5) and (3.6) for each consumer gives a common housing demand function  $\hat{h}(\tilde{r}^i, G, y^i)$ . Again using the homotheticity of the utility function, I can write consumer  $y^i$ 's housing demand in the form

$$\hat{h}(\tilde{r}^i, g, \gamma, y^i) = \phi(y^i)h(\tilde{r}^i, G) = \begin{cases} \phi(y^i)h(\tilde{r}^N, g) & \text{if a nonmember} \\ \phi(y^i - c\gamma)h(\tilde{r}^M, g + \gamma) & \text{if a member} \end{cases}$$

where  $\phi' > 0$ . Note that the  $h$  function is common to members and nonmembers; only the arguments are different. Hence the indirect utility function of each nonmember is

$$V(\tilde{r}^N, g, y^i) = U(y^i - \tilde{r}^N \phi(y^i)h(\tilde{r}^N, g), \phi(y^i)h(\tilde{r}^N, g), g) \quad (3.7)$$

While the indirect utility function of each member is

$$V(\tilde{r}^M, g, \gamma, y^i) = U(y^i - c\gamma - \tilde{r}^M \phi(y^i - c\gamma)h(\tilde{r}^M, g + \gamma), \phi(y^i - c\gamma)h(\tilde{r}^M, g + \gamma), g + \gamma) \quad (3.8)$$

Finally, in this stage, the housing markets clear, and this resolves the net-of-tax housing prices. The two equations that characterize the housing prices are as follows:

$$\int_y^{y^*} \phi(y^i)h((1+t)r^N, g)f(y^i)dy^i = H^S(r^N) \quad (3.9)$$

$$\int_{y^*}^{\bar{y}} \phi(y^i - c\gamma)h((1+t)r^M, g + \gamma)f(y^i)dy^i = H^S(r^M) \quad (3.10)$$

The separability of the housing demand function allows me to pull out the housing demands from the integrals in both equations above. Thus the equilibrium housing price equations are

$$h((1+t)r^N, g)\Phi^N = H^S(r^N) \quad (3.11)$$

$$h((1+t)r^M, g + \gamma)\Phi^M = H^S(r^M) \quad (3.12)$$

$$\text{where } \Phi^N \equiv \int_y^{y^*} \phi(y^i)f(y^i)dy^i$$

$$\text{and } \Phi^M \equiv \int_{y^*}^{\bar{y}} \phi(y^i - c\gamma)f(y^i)dy^i$$

Note that  $\Phi^N$  depends only on  $y^*$  while  $\Phi^M$  depends only on  $y^*$  and  $\gamma$ . These represent aggregate income measures of the nonmembers and the members respectively. Equations (3.11) and (3.12) determine the equilibrium  $r^N$  and  $r^M$  respectively.  $r^N$  depends on  $y^*$ ,  $t$  and  $g$ , while  $r^M$  depends on these parameters and  $c$  and  $\gamma$  as well.

### 3.4 Stage 1: Nash Equilibrium

#### 3.4.1 Stage 1 Equilibrium Without Private Government

Stage 1 without a private government is quite simple. There is no need to determine a marginal member, and of course there is no private government provision. Therefore this stage simply consists of the public government's choice of  $g$ . Now the objective of the public government is to choose provision and taxes to maximize the aggregate welfare of its members, subject to the budget being balanced. The problem is thus:

$$\begin{aligned} \max_{g,t} \quad & \int_{\underline{y}}^{\bar{y}} U [y^i - (1+t)r\phi(y^i)h((1+t)r, g), \phi(y^i)h((1+t)r, g), g] f(y^i) dy^i \\ \text{s.t.} \quad & trh((1+t)r, g)\Phi - cg \geq 0 \end{aligned} \quad (3.13)$$

The constraint simply says that the government's total property tax revenue cannot exceed the total cost of provision. Recall that for both the private and the public governments, I assume that each unit of the public good costs  $c$  of the numeraire to produce. The government's first order conditions (FOCs) for this problem are:

$$\begin{aligned} \text{FOC w.r.t. } g: \quad & \int_{\underline{y}}^{\bar{y}} U_x [-(1+t)r\phi(y^i)h_2] f(y^i) dy^i + \\ & \int_{\underline{y}}^{\bar{y}} U_h [\phi(y^i)h_2] f(y^i) dy^i + \int_{\underline{y}}^{\bar{y}} U_G f(y^i) dy^i + \kappa [trh_2\Phi - c] = 0 \end{aligned} \quad (3.14)$$

$$\begin{aligned} \text{FOC w.r.t. } t: \quad & \int_{\underline{y}}^{\bar{y}} U_x [-r\phi(y^i)h - (1+t)r^2\phi(y^i)h_1] f(y^i) dy^i + \\ & \int_{\underline{y}}^{\bar{y}} U_h [\phi(y^i)rh_1] f(y^i) dy^i + \kappa [rh\Phi + tr^2h_1\Phi] = 0 \end{aligned} \quad (3.15)$$

where  $\kappa$  is the Lagrange multiplier on the constraint, and I have assumed that the budget constraint binds. For clarity I suppress the arguments of the housing function and its partial derivatives. That is,  $h \equiv h(\bar{r}, g)$ ,  $h_1 \equiv \frac{\partial h}{\partial \bar{r}}$  and  $h_2 \equiv \frac{\partial h}{\partial G}$ . Now, if I move the terms involving  $\kappa$  to the other side, and I divide (3.14) by (3.15), I have the following equation after some terms cancel:

$$\frac{trh_2\Phi - c}{rh\Phi + tr^2h_1\Phi} = \frac{\bar{r}h_2\bar{U}_x - h_2\bar{U}_h - \bar{U}_G}{[rh + (1+t)r^2h_1]\bar{U}_x - rh_1\bar{U}_h} \quad (3.16)$$

where  $\bar{U}_x \equiv \int_{\bar{y}} U_x \phi(y^i) f(y^i) dy^i$ ,  $\bar{U}_h \equiv \int_{\bar{y}} U_h \phi(y^i) f(y^i) dy^i$  and  $\bar{U}_G \equiv \int_{\bar{y}} U_G f(y^i) dy^i$ . These are summary measures that define the aggregate marginal utilities of  $x$ ,  $h$  and  $G$  respectively. Because their values differ over the range of incomes, they must remain inside the integrals.

To further simplify equation (3.16), I recall that the Stage 2 problem always implies that  $\frac{U_h}{U_x} = \tilde{r}$ . After substituting out the  $U_h$  terms, I get the following Samuelson condition:

$$c = \Phi \left[ 1 - \frac{t\epsilon_r}{1+t} \right] \left[ \frac{\bar{U}_G}{\bar{U}_x} \right] + \Phi tr \frac{\partial h}{\partial G} \quad (3.17)$$

Here  $\epsilon_r \equiv -\frac{\tilde{r}h_1}{h}$ , the own-price elasticity of housing. This equation, combined with the private government's balanced budget constraint

$$trh(\tilde{r}, g)\Phi - cg = 0 \quad (3.18)$$

characterize the public government's optimal choices of  $g$  and  $t$ , when there is no private government.

Let us examine (3.17) more carefully. This is the modified Samuelson rule for public good provision in the presence of a distortionary property tax. The left-hand side is the marginal rate of transformation between  $G$  and  $x$  — recall it takes  $c$  units of the numeraire to make one unit of the public good. The first term on the right-hand side comprises distortionary factor multiplied by a term clearly representing the sum of the marginal rates of substitution. The second term measures the effect that an extra unit of  $g$  provision has on total tax revenue via the housing market. This second term exists because of the complementarity between the public good and housing services.

What can we say about the size of the terms in equation (3.17)? The term  $\bar{U}_G/\bar{U}_x$  by itself represents the first-best level of provision, which would occur if the public good were financed by a non-distortionary lump-sum tax. Now

$$1 - \frac{t\epsilon_r}{1+t} = 1 + \frac{tr}{h} \frac{\partial h(\tilde{r}, G)}{\partial \tilde{r}}$$

But the Slutsky equation says

$$\frac{\partial h(\tilde{r}, G)}{\partial \tilde{r}} = S_{hh} - h \frac{\partial h(\tilde{r}, G)}{\partial y}$$

where  $S_{hh} < 0$  represents the derivative of the compensated demand for housing. Now because utility is homothetic,  $\frac{\partial h(\tilde{r}, G)}{\partial y} = 0$ . Therefore

$$1 - \frac{t\epsilon_r}{1+t} = 1 + \frac{tr}{h} S_{hh} < 1$$

Hence the “true” benefit of the public good is scaled down from the first-best amount. So with property taxation, this term says there is underprovision of  $g$  relative to the first-best. The magnitude of the scaling-down factor depends not only on the size of the property tax and housing prices, but also on the compensated demand elasticity. If demand for housing is quite inelastic, the scaling-down factor is small. This is because although the property tax raises the price of housing for consumers, it does not cause a great substitution away from housing. Since property tax revenue is relatively unchanged, an amount of  $g$  close to the first-best can be provided. (Indeed, in the limiting case, if housing is perfectly inelastically demanded, this term is 1; since the tax revenue term is positive, the second-best level of  $g$  actually exceeds the first-best level. This will also happen if housing is very inelastically demanded and the tax revenue term is large enough.) On the other hand, if housing is relatively elastically demand, the first term’s effect will tend to dominate the second, leading to a lower level of public good provided.

What about the tax revenue term? If housing and the public good are complements, then provision of  $g$  will increase tax revenue. This will make the second term positive, which means that this distortion term tends to *overprovide*  $g$  relative to the first-best. Clearly, when both distortions are present, it is ambiguous whether the public government with property tax underprovides or overprovides  $g$  relative to the Samuelson rule. The substitutability and elasticity of housing demand are the keys to solving the puzzle.

### 3.4.2 Stage 1 Equilibrium With Private Government

After having examined the implications for a distortionary property tax without a private government, I turn to the situation with a private government. I examine the simultaneous choices of the consumers, public government and private government. This stage will involve the derivation of the three players’ reaction functions. The three reaction functions will characterize the three Nash equilibrium choices of  $y^*$ ,  $g$  and  $\gamma$ , if an equilibrium exists. (The public property tax  $t$  will fall out of the public government’s budget constraint.)

It is important to state that in this stage, all the players take the values of  $r^N$  and  $r^M$  as given. That is, the marginal private government member  $y^*$ , the private government provision  $\gamma$  and the public government provision  $g$  are all determined on the assumption that these choices

will not influence housing prices <sup>8</sup>. I make this admittedly myopic assumption primarily for tractability. If agents take into account the effects their collective actions have on net house prices, the model's complexity would increase substantially. In addition, for the case of private government membership decisions of individuals, it is reasonable to assume that agents do not believe their decisions are great enough to affect house prices. Of course, it is more difficult to justify the same for the public and the private governments. One would think that the private and the public governments' provision choice would be made with the knowledge that they will influence net housing prices.

### 3.4.3 Determination of the Marginal Member

The consumers' choice in Stage 1 is simple. They decide whether or not to join the private government. They take the level of provision of  $g$  and  $\gamma$  as given, as well as the property tax rates and house prices. A consumer of income  $y^i$  will therefore join the private government if and only if the utility of membership exceeds the utility of nonmembership. In other words,  $y^i$  will be a member if and only if

$$U [y^i - c\gamma - \tilde{r}^M \phi(y^i - c\gamma)h^M, \phi(y^i - c\gamma)h^M, g + \gamma] > U [y^i - \tilde{r}^N \phi(y^i)h^N, \phi(y^i)h^N, g] \quad (3.19)$$

where  $h^M \equiv h(\tilde{r}^M, g + \gamma)$  and  $h^N \equiv h(\tilde{r}^N, g)$ , the income-independent housing demand functions. We are looking for a marginal income  $y^*$  for whom (3.19) holds with equality. Since it can be shown that the indifference curves of all individuals are increasing, continuous and concave in the  $(G, \tilde{r})$  plane, then thanks to the single-crossing assumption,  $y^*$  will partition the population into two convex sets: all those  $y \in [\underline{y}, y^*)$  strictly prefer not to join the private government, and all those  $y \in (y^*, \bar{y}]$  strictly prefer to join the private government.

### 3.4.4 Determination of Private Provision

I now move on to the levels of government provision. It is simpler to begin with the choice of the private government. The objective of the private government is to choose provision to maximize the aggregate welfare of its members, subject to the budget being balanced. Because the private government simply charges its members the unit cost of provision, the budget and automatically balanced. So the private government's problem reduces to the following:

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<sup>8</sup>Although residents and governments are not myopic in the sense that residents *will* adjust their housing consumption in response to changes in government provision or taxes.

$$\max_{\gamma, \tau} \int_{y^*}^{\bar{y}} U [y^i - c\gamma - (1+t)r^M \phi(y^i - c\gamma)h((1+t)r^M, g + \gamma), \phi(y^i - c\gamma)h((1+t)r^M, g + \gamma), g + \gamma] f(y^i) dy^i \quad (3.20)$$

The first order condition (FOC) for the private government is:

$$\int_{y^*}^{\bar{y}} U_x^M [-c + \tilde{r}^M \phi'(y^i - c\gamma)ch^M - \tilde{r}^M \phi(y^i - c\gamma)h_2^M] f(y^i) dy^i + \int_{y^*}^{\bar{y}} U_h^M [-c\phi'(y^i - c\gamma)h^M + \phi(y^i - c\gamma)h_2^M] f(y^i) dy^i + \int_{y^*}^{\bar{y}} U_G^M f(y^i) dy^i = 0 \quad (3.21)$$

For clarity I suppress the arguments of the housing function and its partial derivatives. That is,  $h^M \equiv h(\tilde{r}^M, g + \gamma)$ ,  $h_1^M \equiv \frac{\partial h^M}{\partial \tilde{r}^M}$  and  $h_2^M \equiv \frac{\partial h^M}{\partial G}$ . Now, in a similar manner to the base case, I can simplify the first order condition by using the envelope condition from the Stage 2 utility-maximization problem:  $\frac{U_h}{U_x} = \tilde{r}^M$  for all members. I find that the Samuelson condition reduces to the following:

$$c = \frac{\int_{y^*}^{\bar{y}} U_G^M f(y^i) dy^i}{\int_{y^*}^{\bar{y}} U_x^M f(y^i) dy^i} \equiv \frac{\bar{U}_G^M}{\bar{U}_x^M} \quad (3.22)$$

We can clearly see that the absence of a property tax distortion. The private government sets an efficient level of the supplement, corresponding to an aggregate marginal rate of substitution. Note the private government's optimality condition is derived as if it believed that it was the only government. This is because the private government cares only about the effect of providing  $\gamma$  on its members; it does not account for the effect of additional government services on housing decisions outside its gates. And since the public good is financed by a nondistortionary fee rather than a property tax, a first-best level of the private supplement is chosen.

### 3.4.5 Determination of Public Provision

The objective of the public government is to maximize the aggregate welfare of the entire population. This differs from most previous multi-community models of LPG provision because in this model the public government cares about the welfare of members of the private government. The public government cannot deny private government members the consumption of the public good. The problem of the public government is

$$\begin{aligned}
\max_{g,t} \quad & \int_y^{y^*} U [y^i - (1+t)r^N \phi(y^i)h((1+t)r^N, g), \phi(y^i)h((1+t)r^N, g), g] f(y^i) dy^i \\
& + \int_{y^*}^{\bar{y}} U [y^i - c\gamma - (1+t)r^M \phi(y^i - c\gamma)h((1+t)r^M, g + \gamma), \\
& \phi(y^i - c\gamma)h((1+t)r^M, g + \gamma), g + \gamma] f(y^i) dy^i \\
\text{s.t.} \quad & tr^N h((1+t)r^N, g)\Phi^N + tr^M h((1+t)r^M, g + \gamma)\Phi^M - cg \geq 0
\end{aligned} \tag{3.23}$$

Again, I find the first order conditions for the public government's optimization problem, and after some algebra, and using the fact that the Stage 2 optimization by consumers implies that  $\frac{U_h^N}{U_x^N} = \tilde{r}^N$  and  $\frac{U_h^M}{U_x^M} = \tilde{r}^M$ , I get the following:

$$\begin{aligned}
c = \quad & \underbrace{\left\{ \left[ 1 - \frac{t\epsilon_{r^N}}{1+t} \right] r^N h^N \Phi^N + \left[ 1 - \frac{t\epsilon_{r^M}}{1+t} \right] r^M h^M \Phi^M \right\}}_{(i)} \underbrace{\left\{ \frac{\bar{U}_G^N + \bar{U}_G^M}{h^N r^N \bar{U}_x^N + h^M r^M \bar{U}_x^M} \right\}}_{(ii)} \\
& + \underbrace{\Phi^N tr^N \frac{\partial h^N}{\partial G} + \Phi^M \tau r^M \frac{\partial h^M}{\partial G}}_{(iii)}
\end{aligned} \tag{3.24}$$

where in an analogous fashion to the previous section,  $\epsilon_{r^N} \equiv -\frac{\tilde{r}^N h^N}{h^N}$ , the own-price elasticity of housing for nonmembers. Also I define three summary measures of aggregate marginal utility, this time pertaining to nonmembers:  $\bar{U}_x^N \equiv \int_y^{y^*} U_x \phi(y^i) f(y^i) dy^i$ ;  $\bar{U}_h^N \equiv \int_y^{y^*} U_h \phi(y^i) f(y^i) dy^i$ ;  $\bar{U}_G^N \equiv \int_y^{y^*} U_G f(y^i) dy^i$ .

Equation (3.24) is the public government's modified Samuelson rule. This equation, combined with the public government's budget constraint

$$tr^N h((1+t)r^N, g)\Phi^N + tr^M h((1+t)r^M, g + \gamma)\Phi^M - cg = 0 \tag{3.25}$$

characterize the public government's optimal choices of  $g$  and  $t$ .

Let us examine the Samuelson condition (3.24). First, it is important to see that the public government takes the private government's members into its provision decision. This is unsurprising as members contribute to aggregate city welfare just as nonmembers do. Yet because members have an alternative source of public services, they are weighted differently than nonmembers. Starting from the right, term (iii) is similar to the tax revenue effect present in the optimality condition of the private government. Unlike (3.22), however, the public

government takes into consideration the effects of public provision on the housing demands of both members and nonmembers. Again, if housing and government services are complementary enough, these tax revenue distortions imply overprovision of the public service relative to the first-best. A less clear-cut interpretation exists for terms (i) and (ii). It is obviously a weighted MRS, but because members and nonmembers are evaluating the partial derivatives of the utility function at different points, their MRSs are not summable. As a result the terms do not simplify. Rather it looks like a combination of the MRS of members and nonmembers.

### 3.4.6 Resolution of the Nash Equilibrium

I have characterized the equilibrium choices of the three players in Stage 1. Consumers choose their marginal member given the levels of provision by the two governments. The private government and the public government each choose provision, given the marginal member and the provision of the other government. The Nash equilibrium will be a 4-tuple  $\{y^*, \gamma, g, t\}$  such that equations (3.19 with equality), (3.22), (3.24) and (3.25) are all satisfied simultaneously. The subsequent part of the analysis is to determine the conditions in which the equilibrium exists and to demonstrate some of its properties.

## 4 Numerical Results

As the analysis in the previous section is general, it may be difficult to see the properties of the Nash equilibrium without solving the model explicitly. Therefore in this section I work through an example that is solved numerically. In this section I assume consumers share the following Cobb-Douglas utility function:

$$U(x^i, h^i, G) = (x^i)^\alpha (h^i)^\beta (G)^\delta$$

where  $\beta > \delta$  to ensure stability. To get tractable results, I alter the objectives of the public and the private government. I assume majority voting in Stage 1 to determine the levels of  $g$  and  $\gamma$ . This assumption simplifies the Samuelson conditions because it no longer involves aggregating the marginal utilities over groups of consumers.

### 4.1 Stage 2

In Stage 2 the levels of  $g$ ,  $t$  and  $\gamma$  are known and taken as given. The marginal member  $y^*$  is also exogenous. Standard utility maximization implies that nonmembers' demands for housing

and the numeraire are

$$h^N = \frac{\beta}{\alpha + \beta} \frac{y^i}{(1+t)r^N} \quad ; \quad x^N = \frac{\alpha}{\alpha + \beta} y^i$$

while members' demands for these goods are

$$h^N = \frac{\beta}{\alpha + \beta} \frac{(y^i - c\gamma)}{(1+t)r^M} \quad ; \quad x^N = \frac{\alpha}{\alpha + \beta} (y^i - c\gamma)$$

## 4.2 Stage 1

Stage 1 now involves solving for the Nash equilibrium values of  $g$ ,  $t$ ,  $\gamma$  and  $y^*$ . I begin with the marginal member condition. The marginal member  $y^*$  satisfies the following:

$$\begin{aligned} \left( \frac{\alpha^\alpha \beta^\beta}{(\alpha + \beta)^{\alpha + \beta}} \right) (y^* - c\gamma)^{\alpha + \beta} ((1+t)r^M)^{-\beta} (g + \gamma)^\delta \\ = \left( \frac{\alpha^\alpha \beta^\beta}{(\alpha + \beta)^{\alpha + \beta}} \right) (y^*)^{\alpha + \beta} ((1+t)r^N)^{-\beta} g^\delta \end{aligned} \quad (4.1)$$

Isolating  $y^*$ , I get the following condition expression for the marginal private government member:

$$y^* = \frac{c\gamma (r^N)^{\frac{\beta}{\alpha + \beta}} (g + \gamma)^{\frac{\delta}{\alpha + \beta}}}{(r^N)^{\frac{\beta}{\alpha + \beta}} (g + \gamma)^{\frac{\delta}{\alpha + \beta}} - (r^M)^{\frac{\beta}{\alpha + \beta}} (g)^{\frac{\delta}{\alpha + \beta}}} \quad (4.2)$$

Next I derive the Samuelson rule for the level of private government provision. The level of  $\gamma$  is assumed to be chosen in pairwise elections by members of the private government. As such, the median voter theorem can be invoked because the single-crossing condition implies that members' ideal levels of  $g + \gamma$  rises with income. Since  $g$  is taken as given by all members of the private government, the level of  $\gamma$  that prevails is that which is preferred by the median voter in the interval  $[y^*, \bar{y}]$ . I will call this consumer the median voter-member, designated  $\hat{y}$ . Note that the identity of  $\hat{y}$  depends on  $y^*$ . The private government's problem then amounts to maximizing the median voter-member's utility. It is useful to first do this in for a general utility function. The median voter-member's problem is

$$\max_{\gamma} U[\hat{y} - c\gamma - (1+t)r^M \phi(\hat{y} - c\gamma)h((1+t)r^M, g + \gamma), \phi(\hat{y} - c\gamma)h((1+t)r^M, g + \gamma), g + \gamma]$$

The first-order condition is

$$\begin{aligned} U_x^M [-c - (1+t)r^M ch\phi'(\hat{y} - c\gamma) - (1+t)r^M \phi(\hat{y} - c\gamma)h_2] + \\ U_h^M [-ch\phi'(\hat{y} - c\gamma) + \phi(\hat{y} - c\gamma)h_2] + U_G^M = 0 \end{aligned} \quad (4.3)$$

But I now use the envelope condition from Stage 2, which states that for all members,  $U_x^M = \frac{U_h^M}{(1+t)r^M}$ . I am then left with the following first-order condition:

$$c = \frac{U_G^M}{U_x^M} \Big|_{\hat{y}} \quad (4.4)$$

Like the analysis in the previous section, the private government sets a non-distortionary level of  $\gamma$ . This time, the supplement is that which is preferred by the median voter-member. Back to our Cobb-Douglas example, the private government provision condition simplifies to the following:

$$\gamma = \frac{\delta \hat{y}}{c(\alpha + \beta + \delta)} - \left( \frac{\alpha + \beta}{\alpha + \beta + \delta} \right) g \quad (4.5)$$

This is written in the form of a reaction function for  $\gamma$ , and presents a key implication. Note that the negative sign implies that  $g$  and  $\gamma$  are strategic substitutes. As public government spending goes up, the equilibrium private government supplement decreases. However, due to the presence of the distortionary property tax, this decrease is not at a one-for-one rate.

The last problem to solve for in Stage 1 is the public government's provision decision. Here we must consider two cases depending on whether the median voter overall, whom I will call  $\hat{y}$  is or is not a member of the private government.

#### 4.2.1 Case A: $\hat{y} < y^*$

I first consider the case where the median voter overall is not a member of the private government. She chooses  $g$  and  $t$  to maximize utility subject to the community budget constraint and taking  $r^N$ ,  $r^M$ ,  $y^*$  and  $\gamma$  as given:

$$\begin{aligned} \max_{g,t} \quad & \alpha \ln \left( \frac{\alpha}{\alpha + \beta} \hat{y} \right) + \beta \ln \left( \frac{\beta}{\alpha + \beta} \frac{\hat{y}}{(1+t)r^N} \right) + \delta \ln g \\ \text{s.t.} \quad & tr^N \int_y^{y^*} \frac{\beta}{\alpha + \beta} \frac{y^i}{(1+t)r^N} f(y^i) dy^i + \end{aligned} \quad (4.6)$$

$$tr^M \int_{y^*}^{\bar{y}} \frac{\beta}{\alpha + \beta} \frac{y^i - c\gamma}{(1+t)r^M} f(y^i) dy^i = cg \quad (4.7)$$

First off, I simplify the budget constraint. By the fact that

$$\begin{aligned}
cg &= tr^N \int_y^{y^*} \frac{\beta}{\alpha + \beta} \frac{y^i}{(1+t)r^N} f(y^i) dy^i + tr^M \int_{y^*}^{\bar{y}} \frac{\beta}{\alpha + \beta} \frac{y^i - c\gamma}{(1+t)r^M} f(y^i) dy^i \\
cg &= \left( \frac{t}{1+t} \right) \left( \frac{\beta}{\alpha + \beta} \right) \left[ \int_y^{\bar{y}} y^i f(y^i) dy^i - c\gamma \int_{y^*}^{\bar{y}} f(y^i) dy^i \right] \\
cg &= \left( \frac{t}{1+t} \right) \left( \frac{\beta}{\alpha + \beta} \right) [E[y] - c\gamma M] \tag{4.8}
\end{aligned}$$

where  $E[y]$  is the mean income over the whole population, and  $M$  is the population of the private government. After this simplification, the two FOCs for the public government's choices of  $g$  and  $t$  are the following:

$$t = \frac{\delta}{\beta} \tag{4.9}$$

$$g = \frac{\beta\delta}{c(\alpha + \beta)(\beta + \delta)} [E[y] - c\gamma M] \tag{4.10}$$

Note that these choices of  $g$  and  $t$  imply that there is no strategic interaction between the public and the private government, which is plausible as the private government's provision does not affect the median voter's objective function.

#### 4.2.2 Case B: $\hat{y} \geq y^*$

Next, I calculate the public government's provision in the case where the median voter overall is a member of the private government. This is clearly the more interesting case, as this case is the one in which the public and private governments truly interact. This time, the median voter chooses  $g$  and  $t$  to maximize the following utility function subject to the community budget constraint and taking  $r^N$ ,  $r^M$ ,  $y^*$  and  $\gamma$  as given:

$$\begin{aligned}
\max_{g,t} \quad & \alpha \ln \left( \frac{\alpha}{\alpha + \beta} (\hat{y} - c\gamma) \right) + \beta \ln \left( \frac{\beta}{\alpha + \beta} \frac{(\hat{y} - c\gamma)}{(1+t)r^M} \right) + \delta \ln(g + \gamma) \\
\text{s.t.} \quad & tr^N \int_y^{y^*} \frac{\beta}{\alpha + \beta} \frac{y^i}{(1+t)r^N} f(y^i) dy^i + \\
& tr^M \int_{y^*}^{\bar{y}} \frac{\beta}{\alpha + \beta} \frac{y^i - c\gamma}{(1+t)r^M} f(y^i) dy^i = cg \tag{4.11}
\end{aligned}$$

The FOCs for this problem reduce to the following:

$$g = \frac{\delta [E[y] - c\gamma M]}{c(\alpha + \beta)(1+t)} - \gamma \tag{4.12}$$

$$-c\gamma = \frac{t\beta - \delta}{(\alpha + \beta)(1+t)} [E[y] - c\gamma M] \tag{4.13}$$

Note that equation (4.12) is expressed as a reaction function of  $g$  with respect to  $\gamma$ . The slope of the reaction function is clearly

$$\frac{dg}{d\gamma} = \frac{-\delta Y^M}{(\alpha + \beta)(1 + t)} - 1 < -1$$

That is, there is a more than one-for-one reduction in the public government's provision as the private government provides another unit. In other words, the strategic downloading problem is greater than one-for-one. Also, note that in both of the FOCs, the income of the median voter  $\hat{y}$  has disappeared, and instead the relevant income is those of the mean individual. It is interesting that the median voter takes  $M$ , the population of the private government, into account whether he is or is not a member.

### 4.3 Numerical Results

I now present some numerical results of the above model and discuss some of the implications and results. I use the following parameter values for the utility function:  $\{\alpha = 0.3, \beta = 0.5, \delta = 0.2\}$ . As most of the interesting action happens in Stage 1, I will assume that the housing prices are taken as given:  $r^N = 1, r^M = 2$ .<sup>9</sup> I also assume that the cost of provision for both governments is  $c = 1$ . Finally, I need a distribution of exogenous incomes. I use a Weibull distribution, which has a probability density function that is right-skewed (that is, the median is less than the mean). Incomes from this distribution range from zero to infinity. The Weibull distribution has a scale parameter  $\eta$  and a shape parameter  $b$ , and I use the values  $\eta = 10, b = 3$ . This results in a median income of 8.85. The mean income is given by

$$E[y] = \eta \Gamma\left(\frac{1}{\beta} + 1\right)$$

while the population of the private government, which is clearly dependent on  $y^*$ , is

$$M = \exp\left(-\frac{y^*}{\eta}\right)^\beta$$

Finally, the median voter member has income

$$\hat{y} = \eta \left[ \left(\frac{y^*}{\eta}\right)^\beta + \ln 2 \right]^{\frac{1}{\beta}}$$

The parameter values above constitute my benchmark case. I will vary these in order to examine the stability and comparative statics properties of the equilibrium.

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<sup>9</sup>As of now, I take the housing prices as exogenously given. This is because I want to focus on the Stage 1 equilibrium and some of its implications.

I present some preliminary Matlab results in Table 5 and Figure 5. I assume for these calculations that the overall median voter (the one who chooses  $g$ ) is a member of the private government, that is,  $\hat{y} \geq y^*$ . This is the more interesting case as it highlights the interaction between the public and the private government choices. The first column of each panel represents the benchmark parameters. I perform two comparative static exercises: first I increase the price of housing within the private government. Then, I increase the per unit cost of service provision.

The preliminary calculations show that Stage 1 equilibria do exist, and that the equilibrium values behave as theory and intuition suggests. As housing becomes more expensive within the private government, the marginal member increases; that is, fewer people join the private government. As well, voters substitute away from  $\gamma$  and towards  $g$ , which raises the property tax rate. This provides an interesting comparison of the two methods of financing. As the private government housing price increase, the substitution away from  $g$  happens because it reduces the property taxes paid by members. This is replaced by consumption of non-distortionary  $\gamma$ .

When  $c$  is varied, on the other hand the equilibrium marginal member hardly changes, and in fact the change is non-monotonic. However, other variables behave as expected. Both  $g$  and  $\gamma$  decrease, but interestingly the property tax rate increases slightly.

In the near future, I plan to expand upon this numerical analysis by endogenizing the housing prices. Then it will be possible to study whether or not the value of public services would be capitalized into housing prices, and how that would affect Stage 1 equilibrium choices.

## 5 Conclusion

As private governments take over more of the roles previously reserved for public local governments, the strategic interaction between private and public providers of public goods becomes more relevant an area of study. This paper has presented an analysis of a private government in the context of the urban housing market. By incorporating a housing market, another channel is opened for the interaction between private governments and the traditional public government. Numerical analysis showed that public and private government provision are strategic substitutes, and this can result in suboptimal provision of public goods.

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Table 1: **Share of Homeowners Paying Housing Association (HA) Fees**

(All figures are in thousands unless otherwise indicated.)

	Households paying HA fees	Total owner- occupied units	Percentage	Median annual fee
<b>All owner-occupied units</b>	<b>6,448</b>	<b>72,265</b>	<b>8.9%</b>	<b>\$312</b>
<i>New vs Old Units</i>				
New (< 4 yrs) units	1,313	4,690	28.0%	\$372
<i>Region</i>				
Northeast	462	12,987	3.6%	\$504
Midwest	1,120	18,049	6.2%	\$192
South	3,207	26,715	12.0%	\$300
West	1,659	14,514	11.4%	\$432
<i>Urbanicity</i>				
Central City	1,190	16,870	7.1%	\$264
Suburbs	4,795	39,420	12.2%	\$336
Nonmetro	463	15,975	2.9%	\$264

Source: 2001 American Housing Survey - US Bureau of the Census

Table 2: **Share of Homeowners Paying Housing Association (HA) or Condo/Coop Fees**

(All figures are in thousands unless otherwise indicated.)

	Households paying HA or condo fees	Total owner- occupied units	Percentage	Median annual fee
<b>All owner-occupied units</b>	<b>10,209</b>	<b>72,265</b>	<b>14.1%</b>	<b>\$312/\$2016</b>
<i>New vs Old Units</i>				
New (< 4 yrs) units	1,413	4,690	30.1%	\$372/\$1452
<i>Region</i>				
Northeast	1,349	12,987	10.4%	\$504/\$2352
Midwest	1,894	18,049	10.5%	\$192/\$1680
South	4,295	26,715	16.1%	\$300/\$2004
West	2,669	14,514	18.4%	\$432/\$2112
<i>Urbanicity</i>				
Central City	2,361	16,870	14.0%	\$264/\$2268
Suburbs	7,245	39,420	18.4%	\$336/\$1968
Nonmetro	603	15,975	3.8%	\$264/\$1296

Source: 2001 American Housing Survey - US Bureau of the Census

Table 3: **Share of Occupied Units in Gated Communities**

(Units, owned or rented, that report that “Community access is secured with walls or fences.” All figures are in thousands unless otherwise indicated.)

	Units with secured access	Total number of occupied units	Percentage with secured access
<b>All occupied units</b>	<b>7,033</b>	<b>106,262</b>	<b>6.6%</b>
<i>New vs Old Units</i>			
New (< 4 yrs) units	821	5,853	14.0%
<i>Region</i>			
Northeast	691	20,321	3.4%
Midwest	579	24,758	2.3%
South	2,969	38,068	7.8%
West	2,793	23,114	12.1%
<i>Urbanicity</i>			
Central City	2,689	31,730	8.5%
Suburbs	3,986	53,574	7.4%
Nonmetro	358	20,958	1.7%

Source: 2001 American Housing Survey - US Bureau of the Census

Table 4: **Central City–Suburb Differences in California CIDs**

Number of CIDs with reported value					
	Type of CID		Size of CID in units		
	Condos	PDs*	1-25	26-150	151+
<i>Los Angeles Area Counties</i>					
LA City	2878	198	2476	936	171
Rest of LA Co.	4164	764	3899	1393	304
Orange Co.	1367	1174	800	1244	715
Ventura Co.	251	313	123	322	135
Riverside Co.	344	686	208	602	352
San Bernardino Co.	236	286	90	342	122
<i>San Francisco Area Counties</i>					
San Francisco City	1013	26	1248	189	42
Contra Costa Co.	296	457	204	443	190
Alameda Co.	528	459	480	499	166
San Mateo Co.	375	199	299	260	91

Source: Levy and Company, CPAs, 2003

\*PD = Planned Development.

Table 5: **Public Property Tax, Private Fee Financing Numerical Results**

(Incomes are distributed in a Weibull distribution with parameters  $\eta = 10, \beta = 3$ .)

$r^M$	2	1	1.2	1.4	1.6	1.8	2.2	2.4
$y^*$	3.00364	2.64268	2.74809	2.83471	2.90442	2.95983	3.03829	3.06580
$\gamma$	1.78591	1.76841	1.77309	1.77720	1.78068	1.78356	1.78781	1.78935
$g$	0.00857	0.02144	0.01799	0.01497	0.01241	0.01029	0.0072	0.00605
$t$	0.00191	0.00479	0.00402	0.00334	0.00277	0.00230	0.00160	0.00135

$c$	1	1.1	1.2	1.3
$y^*$	3.00364346869	3.00364346863	3.00364346863	3.00364346859
$\gamma$	1.7859	1.6236	1.4883	1.3738
$g$	0.00857	0.00779	0.00714	0.00659
$t$	0.00191029641761	0.00191029641762	0.00191029641772	0.00191029641777

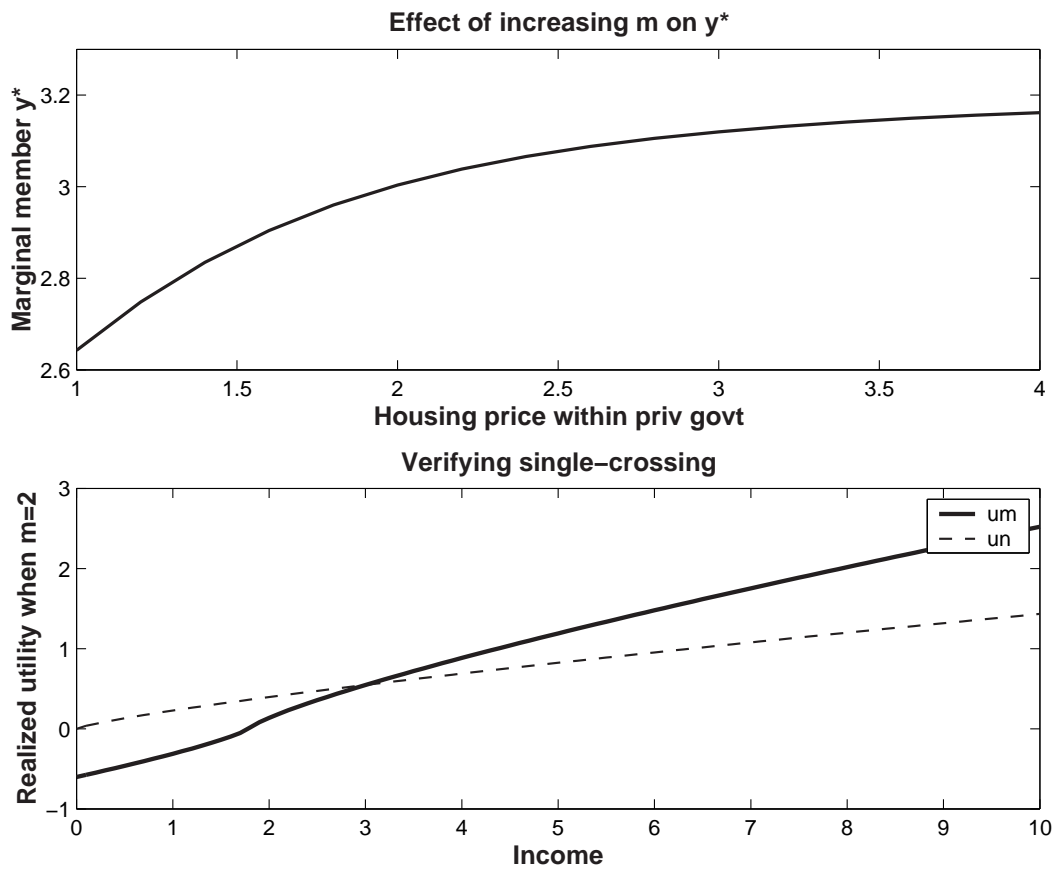


Figure 1: Numerical simulation, Cobb-Douglas utility, majority voting