Octoberfest 2011: Titles and Abstracts

- Steve Awodey: Higher-dimensional inductive types

- Miklós Bartha: Quantum Turing automata and traced monoidal categories

  A directed quantum Turing automaton (DQTA) is defined as a monoidal automaton $(\mathcal{H}, \alpha) : \mathcal{K} \to \mathcal{L}$ over the compact closed category $(\mathbf{FdHilb}, \otimes)$ of finite dimensional Hilbert spaces, where the combinational logic $\alpha : \mathcal{H} \otimes \mathcal{K} \to \mathcal{H} \otimes \mathcal{L}$ is an isometry. Two DQTA are composed by the standard cascade product, and an additive style Turing tensor $\boxtimes$ is defined on such automata using a special blend of $\otimes$ and $\oplus$ in **FdHilb**. By the help of the Moore-Penrose generalized inverse, a totally defined additive trace operation is introduced on the restriction of $(\mathbf{FdHilb}, \oplus)$ to isometries, which coincides with the so called kernel-image partial trace found recently by Malherbe, et. al. for an arbitrary additive category. The additive trace on isometries is carried over to the structure of DQTA, rendering it a traced monoidal category. Using the *Int* construction, this structure is further transformed into the indexed monoidal algebra of undirected quantum Turing automata.

- Robin Cockett: Modeling functional complexity classes categorically
  (Joint with J. Gallagher, P. Hrubes, and J. Ximo)

- Geoff Cruttwell: A tale of two tangent bundles

  There has been recent interest in a generalization of smooth real manifolds to smooth manifolds modeled on locally convex "convenient" vector spaces. However, there is some question about the definition of the tangent bundle in this setting. For smooth real manifolds, there are two equivalent definitions of the tangent bundle. In the first, the tangent bundle consists of small smooth curves into M; in the second, it consists of certain linear functionals on the space of smooth maps from M to R. Interestingly, in the generalized setting of smooth convenient manifolds, these definitions differ.

  In this generalized setting, is there a reason to choose one definition of the tangent bundle over the other? We investigate this question by looking at smooth manifolds via differential categories, tangent structures, and synthetic differential geometry.

- James Dolan: Toposes of toric quasicoherent sheaves

  Analogous to the abelian category of quasicoherent sheaves over an ordinary algebraic variety there's a topos of "toric quasicoherent sheaves" over a toric variety.

- P. Freyd: Mal'cev Categories

- Jeff Egger: Actegories and pro-actegories in theoretical computer science

  Paddy McCrudden coined the term "actegory" to denote a (mere) category C together with an action of a monoidal category (V,.,I) on C. Similarly, we use the term "pro-actegory" to denote a (mere) category equipped with a pro-action of a pro-monoidal category on it. Both actegories and pro-actegories have arisen in modelling effect calculi, and in this talk I will try to explain both how and why.

- Alex Hoffnung: Remarks on Zamolodchikov tetrahedron equations and the Kapranov-Voevodksy construction

  We discuss solutions to Zamolodchikov tetrahedron equations in incidence geometries associated to Hecke algebras. The first goal is to reconcile these solutions with the Z-systems of Kapranov-Voevodsky to construct (weak) 2-braidings. Further, we remark on aspects of a fully weak notion of braided monoidal bicategory as a one-morphism Trimble tetracategory and relations to adjoint properties of spans.

- Rory Lucyshyn-Wright: Totally distributive toposes: Injectivity and sites

  We show that the *lex totally distributive categories* with a small set of generators are exactly the *injective Grothendieck toposes*, studied by Johnstone and Joyal. This result is an extension of an earlier partial result in this regard, given by the speaker in Halifax in 2010. Further, we characterize the totally distributive categories (introduced by Rosebrugh-Wood) with a small set of generators as exactly the *essential* subtoposes of presheaf toposes, studied by Kelly-Lawvere. In view of a result of Kelly-Lawvere, this observation yields a characterization of the totally distributive categories with a small set of generators as the categories of sheaves on a particularly simple class of Grothendieck sites, each of which is specified by a small category $\mathcal{G}$ and an idempotent ideal of arrows in $\mathcal{G}$.

- Gábor Lukács: Universal constructions in topological groups and cofinal types (preliminary report; joint with Boaz Tsaban)

  Let X and Y be posets. We write $X \preccurlyeq Y$ if there is a map $f\colon Y \to X$ such that for every $x \in X$ there is $y_0 \in Y$ such that $f(y) \geq x$ for every $y \geq y_0$. One says that X and Y are *cofinally equivalent* if $X \preccurlyeq Y$ and $Y \preccurlyeq X$.

  Let G be an abelian topological group, and let $\mathcal{N}(G)$ denote the filter of neighborhoods of zero, ordered by reverse inclusion. The *character* $\chi(G)$ is the minimal cardinality of a base at zero for the topology of G, that is, the the minimal cardinality of a cofinally equivalent subset of $\mathcal{N}(G)$. While cardinal invariants of topological groups have been thoroughly studied, their cofinal types seem to have attracted far less attention.

  We are interested in the cofinal types of groups obtained by two universal constructions: coproducts of abelian topological groups, and free abelian topological groups (left adjoint to the forgetful functor $\mathsf{AbTop} \longrightarrow \mathsf{Top}$). Both of them are very simple algebraically, yet immensely complex topologically. In this talk, we recover the topological structure of both constructions using categorical considerations, and present an upper bound for their cofinal types.

- Peter Lumsdaine: Free monads via inductive types.

  An indispensable result of classical category theory is the fact that any endofunctor on a complete category preserving suitably filtered colimits admits free algebras. This provides many desirable free objects; but the standard proof uses transfinite iteration along ordinals, and hence does not automatically transfer to constructive settings.

We recast the proof slightly iterating not along ordinals, but over certain "free filtered categories", which may be constructed directly as inductive-inductive [sic] types, or, with a bit more work, using ordinary inductive definitions. This generalises the theorem to a wide range of constructive settings, and also gives gives a new perspective on the classical case.

- Micah B. McCurdy: A Graphical Treatment of Whiskering in Bicategories

  In general, whiskering oplax or lax natural transformations by lax or colax morphisms in a bicategory is not, in general, well defined. However, John Bourke has shown in an as-yet-unpublished note that they can be whiskered by so-called "separable Frobenius" morphisms of bicategories. We give a graphical proof of his result, and discuss how common such separable Frobenius morphisms are.

- Susan Niefield: Span, Cospan, and Other Double Categories

  Given a double category $\mathbb{D}$ with collages, there is a canonical colax functor from $\mathbb{D}$ to the double category $\mathbb{C}\mathrm{ospan}(\mathbb{D}_0)$ whose objects and horizontal morphisms are the same as those of $\mathbb{D}$ and vertical morphisms are cospans. In this talk, we characterize those double categories for which this canonical functor has a right adjoint. Examples include topological spaces, locales, toposes, small categories, and posets.

- R. Pare: Composing Modules of Lax Functors

  Lax functors of (weak) double categories are a simple generalization of the corresponding concept for bicategories. In this context it is natural ask for horizontal and vertical morphisms of lax functors. The horizontal ones, which we call natural transformations, correspond to Lack's (op)ICONs and present no problem. The vertical ones correspond to the modules of Cockett, Koslowski, Seely, and Wood, and generalize profunctors. Their work shows how,assuming a smallness condition on the domain and some cocompleteness on the codomain, modules can be composed. However their construction doesn't extend immediately to double categories. We generalize and streamline their construction and exhibit a certain factorization condition on cells needed for composition to work well.

- R. Rosebrugh: View updates via a distributive law

  We have shown that when a view functor on database states is a fibration, the view delete updates may be universally lifted to database state updates, and this generalizes the classical "constant complement" updating strategy. We now can show that simultaneous, compatible delete and insert updatability is guaranteed when the view functor is an algebra for the composite monad from a distributive law. The distributive law links the monads for fibrations and opfibrations. Moreover, it is sufficient that the view functor is both a fibration and opfibration and satisfies a Beck-Chevalley condition. (joint with Michael Johnson)

- A. Savage: A graphical categorification of the Heisenberg algebra

  In this talk, we will present a graphical category in terms of certain planar braid-like diagrams. The definition of this category is inspired by the representation theory of Hecke algebras of type A (which are certain deformations of the group algebra of the symmetric group). The Heisenberg algebra (in infinitely many generators), which plays an important role in the description of certain quantum mechanical systems, injects into the Grothendieck group of our category, yielding a "categorification" of this algebra. We will also see that our graphical category acts on the category of modules of Hecke algebras and of general linear groups over finite fields. Additionally, other algebraic structures, such as the affine Hecke algebra, appear naturally.

  We will assume no prior knowledge of Hecke algebras or the Heisenberg algebra. This is joint work with Anthony Licata and inspired by work of Mikhail Khovanov.

- Michael Shulman: Traces in indexed monoidal categories

  An endomorphism of a dualizable object in a symmetric monoidal category has a canonical trace. In algebra, this is the usual trace of a matrix, while in topology, it is the fixed-point index. The Lefschetz fixed-point theorem is an immediate consequence. However, generalizations and converses of the Lefschetz fixed-point theorem use trace-like notions that do not fit the symmetric monoidal framework.

  Kate Ponto showed how to interpret one of these notions, the "Reidemeister trace", using a form of trace in a bicategory. Classically, the fact that the Reidemeister trace refines the fixed-point index is obvious, but it is not so obvious how to compare the two categorical points of view. In joint work with Kate Ponto, we show how to do this using "indexed symmetric monoidal categories" (a.k.a. "symmetric monoidal fibrations"), which give rise to both kinds of trace in a unified context.