

Duality theory of locally precompact groups

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ABSTRACT

A subset B of a (Hausdorff) topological group G is said to be *precompact* (or *bounded*) if for every neighborhood U of the identity in G , there is a finite subset $F \subseteq G$ such that $B \subseteq FU$. Every topological group G admits a completion \bar{G} with respect to its two-sided uniformity, and the completion is unique up to a topological isomorphism. Thus, precompact groups are precisely subgroups of compact groups.

Precompact groups have been in the focus of interest since Comfort and Ross' seminal paper of 1964 on duality theory of abelian precompact groups (cf. [2]). A group G is *locally precompact* if it contains a precompact neighborhood of the identity. The completion \bar{G} of a locally precompact group G is locally compact, and thus such groups are precisely the subgroups of locally compact groups.

The *Pontryagin dual* of an abelian group A is the group \hat{A} of continuous homomorphisms $\chi: A \rightarrow \mathbb{R}/\mathbb{Z}$, equipped with the compact-open topology. By the famous Pontryagin-van Kampen duality, for every locally compact abelian (LCA) group A , the evaluation $\alpha_A: A \rightarrow \hat{\hat{A}}$ is a topological isomorphism.

In this talk, a duality is established between the category LPA of locally precompact abelian groups (and their continuous homomorphisms) and the category KK of compactifications of LCA groups. The dual of G is the compactification $\hat{G} \rightarrow \widehat{G}_d$, where G_d is the group G with the discrete topology. This duality naturally extends that of Pontryagin-van Kampen. Realcompactness and local pseudocompactness are characterized in terms of this dual (cf. [5]).

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