Locally Quasiconnected Toposes

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September 11, 2006

Abstract

We introduce a *geometric* and a *logical* notion of *locally quasiconnected* topos, by weakening, and further weakening, that of a locally connected topos. Each has a distinctive feature which, in the locally connected case, agree [2].

Definition 0.1 Denote by $\mathbf{Loc}_0$ the full subcategory of $\mathbf{Loc}$ whose objects are the 0-dimensional locales, in the sense of the pure sublocales of Stone locales in $\mathscr{S}$ [2] .

1. A topos $f : \mathscr{F} \longrightarrow \mathscr{S}$ is said to be *logically locally quasiconnected (llqc)* if the canonical functor $F^* : \mathbf{Loc}_0 \longrightarrow \mathscr{F}$ has an $\mathscr{S}$-indexed left adjoint $F_!$.
2. A lopos $f : \mathscr{F} \longrightarrow \mathscr{S}$ is said to be a *geometrically locally quasiconnected (glqc)* topos if it is llqc and furthermore the canonically induced $\rho : \mathscr{F} \longrightarrow Sh(F_! 1)$ is a hyperpure surjection [1].

Theorem 0.2 Denote by $\mathbf{T}$ be the full sub 2-category of $\mathbf{Top}_{\mathscr{S}}$ whose objects are the llqc toposes. The pair $(\mathbf{T}, \mathbf{Loc}_0)$ is an instance of a *complete extensive topos doctrine* [3]. The *comprehensive factorization* that arises from it gives an alternative construction of the (hyperpure, Michael complete spread) factorization [1] when restricted to geometric morphisms whose domains $\mathscr{F} \longrightarrow \mathscr{S}$ are definable dominances [2]. The new feature is the involvement of a notion of 0-distribution.

Remark 0.3 Let $\mathbf{T}^*$be the full sub 2-category of $\mathbf{Top}_{\mathscr{S}}$ whose objects are the glqc toposes. The pair $(\mathbf{T}^*, \mathbf{Loc}_0)$ is not an instance of an extensive topos doctrine. On the other hand, the interpretation, for a glqc topos $f : \mathscr{F} \longrightarrow \mathscr{S}$, of the induced $F_! : \mathscr{F} \longrightarrow \mathbf{Loc}_0$ as the "locale of quasicomponents functor", is correct in this case.

(This is work in progress, joint with Jonathon Funk.)

References

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[2] M. Bunge and J. Funk. *Singular Coverings of Toposes*, volume 1890 of *Lecture Notes in Mathematics*. Springer-Verlag, Heidelberg-Berlin-New York, 2006.

[3] M. Bunge and J.Funk. Extensive topos doctrines. In preparation.