WORKSHOP "NEW DIRECTIONS IN INVERSE SEMIGROUPS": ABSTRACTS

Ruy Exel (U. Federal de Santa Catarina, Brazil): Partial group actions and subshifts II
(Minicourse, joint with the "Workshop on Dynamical Systems and Operator Algebras")

Abstract: The first goal of this series of two talks is to present a very general method for describing a C* algebra as a partial crossed product which often applies to C*-algebras generated by partial isometries. This method is based on a paper I wrote with Marcelo Laca and John Quigg many years ago and it has already been used to describe Cuntz-Krieger algebras, Hecke algebras, algebras associated to integral domains, algebras associated to separated graphs and many others.

My second goal is to enlarge the above list by including the Carlsen-Matsumoto C*-algebras associated to subshifts. This class of algebras was introduced by Matsumoto in 1997 and has a very long and rich history, having been intensively studied by many authors. Nevertheless the theory of partial actions can provide new results, notably a complete characterization of simplicity applying to all subshifts, even those which are not surjective. This part of the talk will be based on a joint paper with M. Dokuchaev.

Jonathon Funk (CUNY, US): Isotropy torsors of an inverse semigroup

Abstract: The topos $B(S)$ of an inverse semigroup S is the category of 'étale S-sets' $X \to E$, where $E = E(S)$=idempotents of S. By definition, an isotropy torsor of an inverse semigroup S is a globally supported étale S-set for which the universal action by the isotropy group $Z(E) \to E$ is free and transitive ($Z(E)$=idempotent centraliser). I will explain that there is a bijective correspondence between:

(i) isomorphism classes of isotropy torsors;
(ii) isomorphism classes of étale sections of the isotropy quotient of $B(S)$ in the sense of geometric morphisms;
(iii) central isomorphism classes of homomorphic sections of the maximum idempotent-separating congruence on S, where (let us say) two such sections are centrally isomorphic if one is the conjugate of the other by an order preserving map $t : E \to Z(E)$ such that for all idempotents e, $t(e)^*t(e) = e$.

This explanation illustrates in part not only the natural relationship between inverse semigroups and toposes, but also some aspects of isotropy theory for toposes in general.

[Joint work with Pieter Hofstra and Benjamin Steinberg.]

David Handelman (University of Ottawa): Dimension Groups: their cause and their cure

Abstract: The study of dimension groups and their close (and not-so-close) friends leads to a lot of areas. Among them are C*-algebras, Choquet theory, operator theory, number theory (both algebraic and analytic), probability theory (Markov chains and random walks), algebraic geometry (both real and the usual kind), convex polyhedra, topological dynamical systems (classification), measure-theoretic dynamical systems (classification), representations of compact groups, ..., not to mention such outré subjects as logic and semigroups.

I will discuss a number of problems, mostly concerning positivity, and how they lead to these connections.

Peter Hines (York University, U.K.): Monoids and Categories of Monotone Partial Injections

Abstract: This talk is about some theory relating to inverse semigroups and inverse categories that arises in theoretical and practical computer science – mostly, but not entirely, relating to the field of reversible computation.

Despite its motivation and origins in theoretical computing, this talk is an exploration of the structures involved for their own sake. This turns out to involve interactions between inverse monoids, inverse categories and partially & totally ordered sets.

Simon Henry (École Normale Supérieure, France): On the C^*-algebra of a topos

Abstract: I will explain a new construction that attaches a C^*-algebra to a Grothendieck topos satisfying some topological conditions (a sort of local compactness). In fact one will have both a reduced and a maximal C^*-algebra, a Banach "L1" algebra, and an algebra of compactly supported functions exactly as in the case of topological groupoids. I will also give a brief overview of how Grothendieck toposes relate to some other objects of interest for this workshop (groupoids, inverse semi-groups, quantales...) and how the construction discussed above recovers a lot of examples of classical constructions of C^*-algebras: all C^*-algebras of étale groupoids, graph C^*-algebras and their generalizations, inverse semi-groups C^*-algebras and a large portion of general topological groupoids, convolution C^*-algebras, etc.

Pieter Hofstra (University of Ottawa): Introduction to topos theory

Abstract: In this expository talk I will introduce the basic notions from topos theory and sheaf theory, highlighting key examples from topology and algebra. The aim of the talk is to provide students and researchers from other areas of mathematics with enough intuition for the subject to be able to follow the advanced research talks on the connections between inverse semigroups and toposes.

Ganna Kudryavtseva (University of Ljubljana, Slovenia): Skew Boolean Algebras

Abstract: Skew Boolean algebras are non-commutative generalizations of Boolean algebras and are idempotent counterparts of Boolean inverse semigroups. By means of a non-commutative Stone duality, left-handed skew Boolean algebras correspond to étale spaces over Boolean spaces in a similar way as Boolean inverse semigroups correspond to étale groupoids with Boolean spaces of identities. Skew Boolean intersection algebras are skew Boolean algebras which are also meet-semilattices with respect to the natural partial order and are analogues of Boolean inverse meet-semigroups. Under the non-commutative Stone duality for left-handed skew Boolean algebras, left-handed skew Boolean intersection algebras correspond to Hausdorff étale spaces.

Beside a general overview, I will discuss two recent results: on the structure of free skew Boolean algebras (joint work with Jonathan Leech) as well as on the structure of free skew Boolean intersection algebras and the subtle bijection between their ultrafilters and pointed partitions of non-empty subsets of the generating set. This is parallel to the bijection between ultrafilters of a free generalized

Boolean algebra and non-empty subsets of the generating set and shrinks to this bijection if one additionally imposes the commutativity axiom. Using this bijection, one can express some combinatorial characteristics of finite free skew Boolean intersection algebras in terms of Bell numbers and Stirling numbers of the second kind. Under the canonical inclusion into the k-generated free algebra, where $k \geq n$, an atom of the n-generated free algebra decomposes into an orthogonal join of atoms of the k-generated free algebra in an agreement with the containment order on the respective pointed partitions. For countably many generators, this leads to the ‘partition analogue’ of the Cantor tree whose boundary is the ‘partition variant’ of the Cantor set.

Mark Lawson (i) (Heriot-Watt University, UK): An introduction to Boolean inverse semigroups

Abstract: In this talk, I will introduce the basic definitions of inverse semigroup theory, with an emphasis on the class of Boolean inverse semigroups, and discuss some key examples. I shall produce some lecture notes beforehand and post them on my website (via news) and make them also available to the organizers.

Mark Lawson (ii) (Heriot-Watt University, UK): Boolean inverse monoids, étale groupoids and groups.

Abstract: The goal of this talk is to show how these three apparently different structures are in fact closely related. In addition to discussing the work that colleagues and I have carried out I shall also touch on recent developments by Matui and also by Nekrashevych which I believe can be viewed with profit from the perspective of Boolean inverse monoids.

Wei Lu (University of Ottawa): Coordinatization of MV algebras and some fun things about effect algebras

Abstract: Introduced by C.C. Chang in the 1950s, MV algebras are to many-valued (Lukasiewicz) logics what boolean algebras are to two-valued logic. On the other hand, effect algebras are a class of partial algebras recently introduced by mathematical physicists to describe quantum effects. We first discuss how these two structures are intimately related in the sense that there is a non-full subcategory of effect algebras isomorphic to the category of MV algebras, and look at the construction of coequalizers for effect algebras (by Bart Jacobs) - a task made difficult by the partiality - and use this to characterize the regular monomorphisms. In the second half of the talk, we discuss coordinatization of MV algebras (Lawson & Scott, and also Wehrung) - i.e. MV algebras can be realized as the lattice of principal ideals of boolean inverse semigroups. We give an example of the coordinatization of the rationals in $[0, 1]$ and present a decomposition theorem that generalizes the approach taken, which may be useful for future concrete coordinatization examples.

Daniele Mundici (Università di Firenze, Italy): Geometry of the Lawson-Scott coordinatization of MV-algebras

Abstract: For any countable MV-algebra A the Lawson-Scott coordinatization process picks some unital dimension group (G, u) such that A coincides with the unit interval $[0, u]$ of (G, u), then picks some Bratteli diagram B of (G, u), and finally constructs the inverse semigroup $I(B)$ having the property that A is isomorphic to the MV-algebra of principal ideals of $I(B)$. One may specialize this construction as follows:

(i) let (G, u) be the uniquely determined unital lattice ordered abelian group corresponding to A via the categorical equivalence Γ between MV-algebras and unital lattice ordered abelian groups;
(ii) then let $B = B(A)$ be the uniquely determined direct system of simplicial groups, all with the same unit u, and unit preserving monotone homomorphisms, sitting inside (G, u).

By Marra ultrasimplicial theorem, $\lim B(A), UB(A)$ and (G, u) are isomorphic as unital lattice ordered abelian groups. Via Elliott classification and its K_0-theoretic refinements, the AF-algebra $E(A)$ given by the direct system $B(A)$ satisfies the identity $K_(E(A)) = (G, u)$. The Murray-von Neumann order of projections of $E(A)$ is a lattice. To illustrate the geometry of this special coordinatization process we will exemplify steps (i)-(ii) in the all-important case A when is free, i.e., (by McNaughton theorem), A consists of all continuous piecewise linear continuous functions $f : [0, 1]^n \to [0, 1]$, each linear piece of f having integer coefficients. Our variant of the Lawson-Scott coordinatization process draws from over 65 years of MV-algebraic theory, including the McNaughton representation of free MV-algebras (1951), Chang completeness theorem $MV = HSP([0, 1])$ (1959), Γ functor theory (1986), and the theory of MV-algebraic Schauder bases and their underlying regular/unimodular triangulations of rational polyhedra in euclidean space (1988?).

Jean Renault (Université d'Orléans, France): Semigroups and higher rank graphs

Abstract: The interest here is in dynamical systems which are close to transformation groups, in particular partial group actions, semigroup actions, inverse semigroup actions, and topological higher rank graphs. Under suitable assumptions, their properties are well encoded by a groupoid and a group-valued cocycle. I shall present some examples and applications, in particular with respect to the amenability of these dynamical systems. This talk will be partly based on a joint work with D. Williams.

Pedro Resende (IST Portugal): Stably Gelfand quantales and C*-algebras

Abstract: A *stably Gelfand quantale* is an involutive quantale Q that satisfies

$$aa^*a \leq a \quad \Longrightarrow \quad aa^*a = a$$

for all $a \in Q$. Examples include the quantale $\mathcal{O}(G)$ of an étale groupoid G, and the quantale $\operatorname{Max} A$ of closed linear subspaces of a C*-algebra A. To each projection b of a stably Gelfand quantale Q is associated a complete and infinitely distributive inverse semigroup $\mathfrak{B} \subset Q$ consisting of all the elements $a \in Q$ such that

$$\begin{aligned} a^*a &\leq b\,, \\ aa^* &\leq b\,, \\ ab &\leq a\,, \\ ba &\leq a\,. \end{aligned}$$

Consequently, we also obtain a localic étale groupoid $\mathcal{B}$ via the quantale $\mathcal{O}(\mathcal{B}) = \mathcal{L}^\vee(\mathfrak{B})$. The inclusion $\mathfrak{B} \to Q$ extends to a quantale homomorphism $\mathfrak{b}^* : \mathcal{O}(\mathcal{B}) \to Q$, which, by analogy with the corresponding definition for locales, we regard as the "inverse image homomorphism" of a "continuous map" of quantales

$$\mathfrak{b} : Q \to \mathcal{O}(\mathcal{B})\,.$$

If $Q = \operatorname{Max} A$ for a C*-algebra A, a projection is the same as a sub-C*-algebra B, and we view the pair $(A, \mathfrak{b})$ as a "C*-algebraic bundle" over $\mathcal{B}$. Conversely, any such bundle $(A, p : \operatorname{Max} A \to \mathcal{O}(G))$ yields a sub-C*-algebra $p^*(e) \subset \operatorname{Max} A$, and thus we obtain a general constructive framework that mimics the interplay between Cartan sub-C*-algebras and Fell bundles on étale groupoids and inverse semigroups. Whereas the latter hinges on the existence of faithful conditional expectations and additional conditions such as commutativity and maximality of subalgebras, the language of quantales and inverse semigroups suggests other natural properties of a sub-C*-algebra B. Comparing both sets of properties may be interesting in its own right. For instance, we say that B is *localic* if $\mathfrak{b}$ is a surjection; and that B is *open* if $\mathfrak{b}$ is an *open map* in a sense that generalizes open maps of locales and such that surjections are stable under pullbacks. As an example, Cartan subalgebras in the sense of Renault are localic, which can be proved by showing that any Fell line bundle on a locally compact Hausdorff groupoid G yields a surjection $p : \operatorname{Max} C^*_r(\pi) \to \Omega(G)$). It is not clear whether p is open in general, but in some examples it is, for instance if G is compact. In this talk I will describe this theory to some extent, along with mentioning open questions and related ongoing or future research.

Charles Starling (University of Ottawa): Inverse Semigroups in C^*-algebras

Abstract: This talk is an introduction to the role of inverse semigroup theory in the study of C^*-algebras. Many C^*-algebras of interest are generated by a set of partial isometries closed under multiplication and adjoint - such a set is always an inverse semigroup. Here we give examples of such C^*-algebras, and discuss how properties of the generating inverse semigroup are reflected in the properties of the C^*-algebra. This is meant to be an expository talk, and no knowledge of C^*-algebras is assumed.

Friedrich Wehrung (Université de Caen, France): Type monoids of Boolean inverse semigroups.

Abstract: The type monoid of a Boolean inverse semigroup (BIS) S is the universal monoid of the partial semigroup consisting of the quotient of S, endowed with its orthogonal addition, by its Greens relation $\mathcal{D}$. It bears strong analogies with such different contexts as abstract measure theory (Dobbertins V-measures), lattice theory (the dimension monoid), ring theory (nonstable K-theory). A related analogy is that BISs, endowed with binary operations suitably defined from the multiplication and orthogonal addition, form a congruence-permutable variety [of universal algebras].

The type monoid of a BIS is a conical refinement monoid, and further, due to Dobbertin's results on V-measures, the converse holds in the countable case. On the other hand, there are counterexamples, due to the author, in cardinality $\aleph_2$. By using a small fragment of the dimension monoid in lattice theory, one can see that the positive cone of any abelian lattice-ordered group is isomorphic to the type monoid of a BIS. We survey some further results of that sort, and we relate the type monoid of S and the nonstable K_0-theory of a K-algebra denoted by $K(S)$, for a unital ring K, observing in particular that even for K a field, the canonical map between those monoids can fail to be an isomorphism.