ECO 6120
Macroeconomic Theory IV
SERGE COULOMBE
EXAM 1

May 24th 2016, 10 AM to 12.45 PM.
Note: the exam is out of 100 points. The student is allowed to bring a hand-written memory aid that stands on one double-sided 8 ½ x 11 page.

Answer each of the following questions in the examination booklet.

1. (35 points) Consider a benevolent central planner in the Ramsey growth model without technological progress but with population growth. The problem of the planner is to choose a time path for \( c \) that maximizes \( U(0) \) in equation (1) subject to the equation of motion of \( k \) in (2), for a given \( k(0) \), and for \( c \geq 0 \) and \( k \geq 0 \). Assume a standard neo-classical production function \( y = f(k) \).

\[
U(0) = \int_{0}^{\infty} e^{-(\rho-n)t} \left[ \frac{c(t)^{1+\theta} - 1}{1-\theta} \right] dt \quad (1)
\]

\[
\dot{k} = f(k) - c - (n+\delta)k \quad (2)
\]

a) Explain in words (one or two sentences for each) what variables and parameters \( c, k, \dot{k}, n, U(0), \rho \) and \( \theta \) stands for. (7 points).

b) Write downs and explain the present-value Hamiltonian for this optimization problem. Provide an economic interpretation of the Hamiltonian (5 points).

c) Derive the solution for \( \frac{\dot{c}}{c} \) for this optimization problem step by step. Explain what you are doing (8 points).

d) Illustrate the steady state solution using the phase diagram. Write down the equations for the two curves. Explain briefly (5 points).

e) Suppose that at time \( t_1 \) we have a once and for all unexpected decrease in the rate of population growth. What happens to \( c \) and \( k \) in the short, the medium, and the long run? Explain your analysis using the Ramsey phase diagram. (10 points)
2. (15 points) Consider Barro (1987) analysis of the effect of a temporary increase in government expenditure (seven years’ war) in the Ramsey growth model. Suppose that government expenditures do not enter the utility function or the production function. Suppose there is no technological progress.

a) As in Barro’s analysis suppose that increase in \( G \) is unanticipated and that it is expected to be temporary (seven years). Using Ramsey’s phase diagram, analyze the impact of the temporary increase in \( G \) on the path of consumption and the capital labor ratio. Draw three diagrams describing the time path of consumption, the capital/labor ratio, and the interest rate. (8 points).

b) Suppose now that after 4 years, the war ends abruptly and again unexpectedly (complete surprise, up to that point people were certain the war will end after seven years)). What will happen to consumption and the capital/labor ratio. Illustrate in the phase diagram and explain. (7 points)

3. (10 points) In economic growth models, technological progress is a necessary condition for having increasing living standards in the long run. True, false, or uncertain? Explain using the tools developed in the course. Maximum 1 page.

4. With the standard Cobb-Douglas production function \( Y = K^\alpha (AL)^{1-\alpha} \), and assuming that the level of technology \( A \) is the same across countries, the Solow growth model is not able to account for size of differences in living standards observed between rich and poor countries. Explain algebraically. What is the solution to this problem proposed by Mankiw, Romer, and Weil (1992)? Explain briefly (10 points).

5. (10 points) Starting from the following production function: \( Y_i = K_i^\alpha (A_iH_i)^{1-\alpha} \), explain in detail the methodology used by Hall and Jones (1999), as explained in Romer textbook, to account for cross-country differences in living standards. How does Hall and Jones measure differences in human capital? What are their results? Explain in detail.
6. (20 points) Consider the AK growth model with consumer optimization. The representative household seeks to maximize the intertemporal utility function:

\[ U(0) = \int_0^\infty e^{-(\rho-n)t} \ln[c(t)] \cdot dt, \]

Under the following constraint: \[ \dot{a} = w + (r - n)a - c \]

Initial wealth \( a(0) \) is given and Ponzi scheme is not allowed.

Output per capita \( y = Ak \) and the rental price of capital equals \( r + \delta \)

a) With this utility function, what is the elasticity of marginal utility (the coefficient of relative risk aversion)? (2 points, no demonstration required)

b) Write down the Hamiltonian for this specific problem and derived algebraically the solution for \( \hat{c}/c \) from the first-order conditions. Explain briefly. (9 points)

c) Suppose that firms profit maximization yields to equalization of the rental price of capital \( r + \delta \) with the marginal product of capital, hat will be the solution for \( \hat{c}/c \)? Explain briefly. (4 points)

d) Suppose that there is a sudden decrease in \( \rho \), what happen to the time path of \( c \)? Illustrate graphically the time path of \( \ln(c(t)) \) through time and explain. (5 points)