Surveying short run and long run stability issues with the Kaleckian model of growth

by

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Introduction

Writing a survey on the Kaleckian model of growth and distribution is a difficult task in view of the existence of the excellent survey that has already been provided by Blecker (2002). Since then, another survey, just as complete, has been written in French by Allain (2008). In addition, at least three other chapters in the present book deal with complications involving the Kaleckian model. As a result the present paper will deal with elementary issues of stability, both in the short run and in the long run. We start with the former.

2. The standard Kaleckian model

The usual Kaleckian model is made up of three equations: an investment equation, a saving equation, and a pricing equation. Each of these equations can be made more complicated at will, as will be shown in other chapters, and of course we may wish to add other equations, for instance equations defining inflation determination (Cassetti, 2002; Lavoie, 1992, ch. 7), or central bank reaction functions (Lavoie and Kriesler, 2007). Here we stick to the basic model.

(1) \( r = \mu u / \nu \)
(2) \( g^s = sp_r \)
(3) \( g^i = \gamma + \gamma_u u + \gamma_m \)

We assume away overhead labour (but see Rowthorn (1981) and Lavoie (1992)), so that the pricing function in terms of the profit rate \( r \) depends simply on the profit share \( m \), the rate of capacity utilization \( u \), and \( v \) the capital to capacity ratio. The higher \( m \), the lower the real wage. We assume no saving out of wages, so that the saving function in growth terms depends only on the profit rate and the propensity to save out of profits \( sp \).\(^1\)

Finally there is the contentious issue of the investment function. We adopt a linear variant of the popular Bhaduri and Marglin (1990) function, which can also be attributed to Kurz (1990), so that investment depends on some constant, the rate of capacity utilization, and the share of profit (or more likely on the normal profit rate, as we shall see later), with \( \gamma \), \( \gamma_u \) and \( \gamma \) being three parameters. The advantage of this investment function, as is now well-known, is that it provides for richer possibilities. In addition, the function can be easily tested empirically, since statistics on both the rate of utilization and the profit share can easily be obtained. A drawback of this function is that it is not really clear why investing entrepreneurs would care about the profit share, in contrast to the profit rate, which is usually the third component of the canonical Kaleckian growth model. In addition, in a model with overhead labour, the profit share is an endogenous variable, which depends on the rate of utilization.

A way out is to argue, from a purely theoretical standpoint, that what Bhaduri and Marglin really have in mind is that investment depends on expected profitability,

\(^1\) We could assume that there is consumption out of wealth, but this would barely change things, as the saving function would become: \( g^s = sp_r - c_w \), with \( c_w \) the propensity to consume out of wealth. But it shows that the saving function need not arise from the origin.
computed at normal prices based on the normal rate of capacity utilization, a point made frequently by Sraffian authors such as Ciconne (1986, p. 26), Vianello (1989), and Kurz (1990). This expected profitability at the normal rate of capacity utilization, which we call $u_n$, is the normal profit rate, which we denote as $r_n$. We may thus rewrite equations (1) and (3) in a way which is amenable to this reinterpretation:

\[ (1A) \quad r = (r_n/u_n)u \]
\[ (3A) \quad g' = \gamma + \gamma_u u + \gamma_r r_n \]

Obviously, $r_n = mu_n/v$, and it makes little difference to use one or the other formulations.\(^2\) Combining equations (1), (2), and (3) to obtain the equilibrium rate of utilization, we get:

\[ (4) \quad u^* = \frac{\gamma + \gamma_m}{s_p (m/v) - \gamma_u} \]

Whereas combing equations (1A), (2) and (3A), we obtain:

\[ (4A) \quad u^* = \frac{\gamma + \gamma_r r_n}{s_p (r_n/u_n) - \gamma_u} \]

To make economic sense $u^*$ must be positive. Hence, if the denominator is positive, its numerator must also be positive; and if the denominator is negative, the numerator must be negative, implying, since $\gamma, m$ is necessarily positive, that the $\gamma$ parameter must be negative and its absolute value sufficiently large.\(^3\) These conditions will play a role in our analysis of stability.

3. Some preliminaries

The first issue we wish to tackle is that of short-run stability. For that problem to exist, there must exist some discrepancy between aggregate demand and aggregate supply, or at least intended aggregate demand and supply. There are a few ways out.

(a) We may suppose that output, or capacity utilization, is given, and that an adjustment occurs within the period, through changes in profit margins, or changes in prices at given nominal wage rates. In this case aggregate demand immediately adapts to aggregate supply. This is sometimes associated with a so-called ultra-short or market period. Some say that this is what Keynes had in mind in some passages of the General Theory (Dutt, 1987; Hartwig, 2007). This mechanism can be found in very few heterodox

\(^2\) If one considers that equation (3A) is the most correct investment equation, then equation (3) should really be rewritten as: $g' = \gamma + \gamma_u u + \gamma_r (mu_n/v)$. But we will leave it at that.

\(^3\) Although this is a trivial point, both Blecker (2002, p. 137) and Bruno (1999, p. 135) draw unwarranted restrictions by forgetting that the $\gamma$ parameter could be negative. Lavoie (1992, pp. 341-3) shows that a negative $\gamma$ can also enrich the range of possible results in a model with overhead labour.
works (e.g., Skott, 1989). It is also the standard Walrasian adjustment mechanism. We assume away this mechanism.

(b) By contrast, we may suppose that the short period is sufficiently long for firms to change output and capacity utilization in line with aggregate demand. In this case, it is now aggregate supply that very quickly adapts to aggregate demand. The adjustment is a pure quantity adjustment. This is the standard interpretation of Keynes, and it is sometimes considered to be his key contribution (Leijonhufvud, 1968, p. 52). Keynesian and Kaleckian authors usually make use of this assumption in their models, and for this reason Duménil and Lévy (1987, p. 136) call it the Keynesian adjustment process. But because aggregate supply is being equated to aggregate demand in each and every period, they also call these models, equilibrium dynamics models.

(c) Finally, another possibility is to assume no market clearing in the short period. Ideally, one should then take into account the evolution of inventories and their impact on rates of capacity utilization as firms try to bring them back to their normal levels (Duménil and Lévy, 1987, 1993; Godley and Lavoie, 2007). But a less demanding strategy is to assume that the adjustment towards aggregate demand and supply equality is only gradual, and is being done through changes in both profit margins and rates of capacity utilization, without keeping track of the inventories. This is what we shall do here.

4. The pure Keynesian adjustment process

As a start, let us consider the pure Keynesian adjustment process, the so-called equilibrium dynamics. To do so, let us distinguish between the realized rate of capacity utilization $u$ and the expected rate of capacity utilization $u^e$, that is, the rate of capacity utilization that entrepreneurs expect to realize in the current period when supply responds to demand. We may presume that entrepreneurs will invest in the current period as a function of the share of profits (the normal profit rate) and the rate of capacity utilization that they expect to be realized as firms modify output in response to sales. In this case, the investment function needs to be slightly modified to:

$$g^i = \gamma + \gamma_u u^e + \gamma_r m$$  \hfill (3B)

Thus now the investment function depends on the expected rate of capacity utilization whereas the saving function depends on the realized rate of capacity utilization, which, combining equations (1), (2), and (3B), is given by:

$$u^K = \frac{\gamma + \gamma_r m + \gamma_u u^e}{s_p (m/v)}$$  \hfill (4B)

We denote by $u^K$ this short-period equilibrium rate of capacity utilization, to indicate its Keynesian or Kaleckian pedigree. Visually, two cases can be distinguished, depending on the slopes of the investment and saving functions. As we shall see, Figure 1 corresponds to the case of Keynesian stability, or stability in dimension as Duménil and Lévy (1993) like to call it, with the slope of the investment function being smaller than
that of the saving function. The Keynesian stability condition holds when the following inequality is verified:

\[(5) \quad \gamma_u - \frac{s_p m}{v} < 0\]

**INSERT FIGURES 1 AND 2**

On the basis of the expected rate of capacity utilization, firms engage in investment expenditures corresponding to \(g'(u^e)\). At that level of capital accumulation, and with the given propensity to save out of profits, aggregate demand will be such that sales will induce a rate of capacity utilization equal to \(u^K\) – the short-run Keynesian equilibrium – as shown in Figure 1. A very similar process is described with the help of Figure 2 that corresponds to the case of Keynesian instability, and where the slope of the investment function is larger than that of the saving function, such that:

\[(5A) \quad \gamma_u - \frac{s_p m}{v} > 0\]

Why does Figure 1 illustrate Keynesian stability whereas Figure 2 illustrates Keynesian instability? With adaptive expectations about capacity utilization, the evolution of the expected rate of capacity utilization is described by the following differential equation:

\[(6) \quad \Delta u^e = \theta(u^K - u^e)\]

In Figure 1, the expected and the realized short-run rates of capacity utilization will converge towards the equilibrium rate of capacity utilization \(u^*\), as entrepreneurs realize that they were overly optimistic. In Figure 2, entrepreneurs overestimate the equilibrium rate of capacity utilization \((u^K > u^*)\), but the realized short-run rate of utilization is even higher than the overestimated rate \((u^K > u^e)\), so that entrepreneurs are induced to raise the expected rate of utilization still further more, thus moving away from the equilibrium defined by equation (4).

Thus, if we adopt the pure Keynesian adjustment mechanism, condition (5) must hold, unless other dynamic adjustment mechanisms are put in place.

**5. A dual adjustment process**

**Questioning Keynesian stability**

It has been recently argued by some post-Keynesian authors, most notably Dallery (2007) and Skott (2008), that the Keynesian stability condition was unlikely to be met, for calibrated values of the main parameters of the model. This can be readily seen. The main problem is that for utilization rates and growth rates to move within a reasonable range of values, one needs the \(\gamma_u\) parameter in equation (3) to be around 0.30, for in that case, for instance, rates of utilization moving between 75 and 85%, as they have done historically, will generate growth rates moving up by 3%, say between 1% and 4% – a range of values that has been observed within industrialized economies. But now the problem is that,
even with generous estimates, \( s_p = 0.8, m = 0.4, \) and \( v = 2 \), the term \( s_pm/v \) is no higher than 0.16 (or if we look at equation (4A), the term \( r_n/u_n \), with an 80% rate of capacity utilization, would imply a profit rate of 20%). A possible answer would be to say that the saving of workers has been omitted, and that adding this saving component would help salvage a modified stability condition, as the saving equation would include an additional term that is sensitive to changes in the rate of utilization, helping to fulfill the stability condition. One would have:

\[
(2A) \quad g^s = s_pm/v + s_w(1-m)u/v
\]

where \( s_w \) is the propensity to save out of wages, with \((1-m)\) being the share of wages.

**Getting away from the equilibrium dynamic model**

But whether or not this assessment is correct, is there any way for Kaleckian models to retain stability, despite the failure of the Keynesian stability condition – equation (5) – to hold? One possibility has been explored by Bruno (1999) and Bhaduri (2006, 2008), and is the subject of this section.

Both Bruno and Bhaduri start away from the equilibrium dynamic model, assuming the absence of market clearing in the short period. Thus, in the short (or ultra-short) period, (intended) investment and saving are not equal. Capacity utilization is fixed, as are profit margins. But let us assume that both quantities and prices react to disequilibria, so that two adjustment mechanisms get going simultaneously, as shown in equations (7) and (8):

\[
(6) \quad \Delta u = \mu(g' - g^s) \quad \text{with} \quad \mu > 0
\]

\[
(7) \quad \Delta m = \psi(g' - g^s)
\]

Equation (6) represents the quantity adjustment mechanism. Firms increase capacity utilization whenever investment surpasses saving, that is, whenever output demand is above production. Equation (7) is the price adjustment mechanism. One would presume that the \( \psi \) parameter is necessarily positive. When output demand is above production \((g' > g^s)\), prices and profit margins rise, thus leading to a rise in the profit share \( m \) (or in the normal profit rate \( r_n \)). This case corresponds to the standard classical price adjustment mechanism, and it also corresponds to the Cambridge adjustment mechanism, found in the earlier post-Keynesian growth models à la Kaldor and Robinson, and associated with forced saving. Bhaduri (2008) however argues that the alternative, with \( \psi < 0 \), is not inconceivable. With excess demand, firms must raise rates of capacity utilization and hence employment rises faster than capacity, and this may generate a stronger bargaining position for workers, as argued in particular by Goodwin (1982). Thus, under some circumstances, when output demand is above production, it may be that real wages rise and hence that profit margins and the profit share \( m \) falls. We shall call this the Radical case (or the Goodwin case), since this kind of profit-squeeze behaviour has been underlined mostly by Radical economists.
We can compute the partial derivatives of this system of differential equations, given by equations (6) and (7), with the help of equations (1), (2), (3). Omitting the constant terms, these partial derivatives can be put in matrix form (matrix $J$) as:

$$
\begin{bmatrix}
\frac{d\Delta u}{d\Delta m} =
\begin{bmatrix}
\mu (\gamma_u - s_p m/v) & \mu (\gamma_r - s_p u/v) \\
\psi (\gamma_u - s_p m/v) & \psi (\gamma_r - s_p u/v)
\end{bmatrix}
\begin{bmatrix}
\Delta u \\
\Delta m
\end{bmatrix}
\end{bmatrix}
$$

An examination of matrix $J$ will be sufficient to determine whether the system is locally stable or not. The determinant of the matrix is zero, implying that this system has a zero root and hence that there is a multiplicity of equilibria on a single demarcation line. Whether this locus of equilibria is stable or not depends on the sign of the trace of the matrix. The model as modified is stable whenever the trace is negative, and it is unstable whenever the trace is positive. The trace of the matrix is equal to the sum of the two diagonal terms:

$$
\text{Tr } J = \mu (\gamma_u - s_p m/v) + \psi (\gamma_r - s_p u/v)
$$

Keynesian stability, or stability in dimension, requires that the first term of the trace be negative. Given that $\mu > 0$, it means that equation (5) is verified, as in the Keynesian adjustment process. Stability in proportion requires that the second term, associated with changes in profit margins, be negative. In the case of the classical or Cambridge adjustment process (with $\psi > 0$), this will occur whenever investment does not react too briskly to changes in profit margins, that is, when:

$$
\gamma_r - s_p u/v < 0
$$

When equation (9) is verified, excess demand leads to an increase in profit margins and profit shares, with a moderate positive impact on investment, and a more important impact on saving, thus bringing together saving and investment, and thus bringing the economy towards equilibrium – a point made early on by Pasinetti (1962). With no quantity adjustment (with $\mu = 0$), this process through price adjustment guarantees the stability of the system.

By contrast, in the Radical case, with $\psi < 0$, excess demand leads to a fall in profit margins and profit shares. To reduce the discrepancy between investment and saving, investment must react strongly to the fall in the profit share, decreasing faster than saving does, and thus in this alternative case, stability in proportion requires that equation (10) be fulfilled.

$$
\gamma_r - s_p u/v > 0
$$

With both the quantity and the price mechanisms in action, no less than 8 cases, all shown in Table 1, become possible. With stability in both dimension and proportion, the trace is necessarily negative, and stability is unconditional. Symmetrically, with instability both in dimension and proportion, the trace is necessarily positive, and the model unstable. In the other four cases, stability is conditional. Thus, in the absence of Keynesian stability, the Kaleckian growth model may still be stable.
### Table 1

<table>
<thead>
<tr>
<th>Sign of $\gamma_u - s_p m/v$</th>
<th>Sign of $\gamma_r - s_p u/v$</th>
<th>Classical or Cambridge case: $\psi &gt; 0$</th>
<th>Radical case: $\psi &lt; 0$</th>
<th>$\frac{du/dm}{\gamma_r - s_p u/v}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(−) Stability in Dimension</td>
<td>(−)</td>
<td>(A) $\psi(\gamma_r - s_p u/v) &lt; 0$</td>
<td>(B) $\psi(\gamma_r - s_p u/v) &gt; 0$</td>
<td>(−) Wage-led locus</td>
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<td></td>
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<td>Stability in proportion $\text{Tr} J &lt; 0$</td>
<td>Instability in proportion $\text{Tr} J &gt; 0$</td>
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<td></td>
<td></td>
<td>Unconditional stability</td>
<td>Conditional stability if $\mu$ is large</td>
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<td>(+)</td>
<td>(C) $\psi(\gamma_r - s_p u/v) &gt; 0$</td>
<td>Instability in proportion $\text{Tr} J = ?$</td>
<td>Stability in proportion $\text{Tr} J &lt; 0$</td>
<td>(+) Profit-led locus</td>
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<td></td>
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<td>Conditional stability if $\psi$ is large</td>
<td>Unconditional stability</td>
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<td>(−)</td>
<td>(E)</td>
<td>$\psi(\gamma_r - s_p u/v) &lt; 0$</td>
<td>(F) $\psi(\gamma_r - s_p u/v) &gt; 0$</td>
<td>(−) Profit-led locus</td>
</tr>
<tr>
<td>Instability in Dimension</td>
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<td>Instability in proportion $\text{Tr} J &gt; 0$</td>
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<td>Conditional stability if $\psi$ is large</td>
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<td>(+)</td>
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<td>$\psi(\gamma_r - s_p u/v) &gt; 0$</td>
<td>(H) $\psi(\gamma_r - s_p u/v) &lt; 0$</td>
<td>(+) Profit-led locus</td>
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<td>Instability in proportion $\text{Tr} J &gt; 0$</td>
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<td>(−) Wage-led locus</td>
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</table>

#### Profit-led and wage-led regimes

Table 1 also highlights the fact that, using the terminology of Blecker (2002), whether the economy is wage-led or profit-led in terms of aggregate demand, that is, relative to the rate of utilization, depends on the signs of the first two columns. This can be seen by taking the total differentials of the combination of equations (1) and (2), and of equation (3):

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4 Bhaduri and Marglin (1990) call these stagnationism and exhilarationism regimes, while Kurz (1990) uses the expressions underconsumption and supply-side regimes.
Equating the above two equations, we obtain the equation which is in the last column of Table 1. Thus, unless we have a priori opinions about the values taken by the parameters in the investment and saving functions, a wage-led aggregate demand regime is as likely as a profit-led regime. \(^5\) It is interesting to note that some configurations are impossible. For instance, a stable wage-led (in aggregate demand or in growth) economy with a classical or Cambridge price adjustment mechanism is only compatible with stability in dimension. Similarly, a stable profit-led economy with a Radical or profit-squeeze price adjustment mechanism requires stability in dimension.

**Graphical illustrations of the dynamics**

Figures 3 to 6 illustrate the transition dynamics in the various cases. Figures 3 and 4 illustrate the Keynesian or dimension stability cases. When there is excess demand, the rate of utilization rises, and this will tend to bring the economy towards the equilibrium locus in the Keynesian stability case. When the economy is wage-led, the addition of the Cambridge price adjustment mechanism (rising profit margins with excess demand) will reinforce this tendency, as shown with the A arrow in Figure 3 (which corresponds to the A entry in Table 1). But with a Radical price adjustment mechanism (falling profit margins with excess demand), stability may either occur (arrow B\(_S\)) or not occur (arrow B\(_U\)). When the economy is profit-led, the reverse occurs. With the addition of the Cambridge price adjustment mechanism, convergence may either occur (arrow C\(_S\)) or not occur (arrow C\(_U\)), whereas it will always occur with the addition of a Radical price adjustment mechanism (arrow D).

**INSERT FIGURES 3 TO 6**

Keynesian instability is illustrated in Figures 5 and 6. This time, when there is excess demand, increases in rates of utilization are driving away the economy from the equilibrium locus. When the economy is profit-led, the addition of a Cambridge price adjustment mechanism provides for conditional stability (arrows E), whereas the addition of a Radical mechanism makes the model completely unstable (arrow F). With a wage-led regime, it is the Cambridge price adjustment mechanism that will make the model unconditionally unstable (arrow G). With a Radical mechanism, convergence may either arise (arrow H\(_S\)) or not occur (arrow H\(_U\)).

It could be interesting to link these disequilibrium dynamics to the standard representation of the Kaleckian growth model. This is done in Figures 7 and 8. Figure 7 illustrates the Keynesian stability case. In the initial steady state, the rate of capacity

\[
\begin{align*}
\frac{dg^s}{dm} &= s_p(m/v)du + s_p(u/v)dm \\
\frac{dg'}{dm} &= \gamma_u du + \gamma dm \\
\end{align*}
\]

\(^5\) With the same two differential equations, one can also assess the conditions under which the economy is in a wage-led growth regime or a profit-led growth regime. As one would expect, in the case of Keynesian stability, a wage-led growth regime is more likely when investment is mainly sensitive to utilization rates and less so to profit shares. The sign of \(dg/dm\) depends on the following expression:

\[
\frac{\gamma_u u - \gamma_r m}{\gamma_u - s_p m / v}
\]
utilization is given by $u_0$. We then assume an upward shift in the $\gamma$ parameter of the investment function, so that the investment curve $g^i$ shifts up, so that, now we have $g^i > g^s$. With the pure Keynesian adjustment process, the economy would move to a new steady state, at the higher rate of utilization $u_1$. This rate of utilization is the same rate $u_1$ that can be found in Figures 3 and 4. However, with the dual adjustment process, profit margins will change. Assuming a classical or Cambridge adjustment process, profit margins and profit shares go up, so that the saving function rotates upwards while the investment function shifts up. In the case of the wage-led economy, with $\gamma_r - s_p u/v < 0$, the shift in the investment function will be small relative to the shift in the saving function, so that the new equilibrium will be $u_{wl}$, below the equilibrium $u_1$ that would have existed without the increase in profit margins. This corresponds to the $u_{wl}$ point found in Figure 3. In the case of the profit-led economy, with $\gamma_r - s_p u/v > 0$, the shift in the investment function will be relatively large, so that the new equilibrium will be $u_{pl}$, above the equilibrium $u_1$ that would have existed without the increase in profit margins. This corresponds to the $u_{pl}$ rate found in Figure 4.

**INSERT FIGURES 7 AND 8**

A similar exercise can be conducted with Keynesian instability, illustrated with Figure 8. The economy, initially, stands at $u_0$. There follows a positive shock on the investment function, shifting the investment curve upwards. With no change in profit margins, the new equilibrium ought to be at $u_1$. However, this could only be a virtual equilibrium, for no economic forces will drive the economy towards it. On the basis of the Keynesian adjustment, $u_1$ is not a stable equilibrium, because, with $g^i > g^s$ at the initial rate of utilization $u_0$, the rate of utilization tends to rise, moving away from $u_1$. However, with the addition of a classical price adjustment mechanism, the economy converges conditionally towards a new equilibrium, at the rate of utilization $u_{pl}$ for instance. This rate corresponds to the rate $u_{pl}$ of Figure 5. In this case, the economy is profit-led, because a higher rate of utilization is associated in equilibrium with a higher profit margin.

**6. Kaleckian in the short run, classical in the long run?**

Several economists would argue that, so far, the analysis has been confined to the short and medium runs, or to provisional equilibria, as Chick and Caserta (1997) would call them. In the long run, critics of the Kaleckian model would say, two things are likely to happen. First, the rate of utilization should come back to its normal value. Secondly, the actual rate of growth of the economy should approximate the natural rate of growth, for otherwise the rate of unemployment would keep rising or falling without limits, but that issue is handled somewhere else in the book (Dutt, 2009). The latter problem has been brought up and tackled more recently; the former problem has been noted more than 20 years ago (Kurz, 1986; Committi, 1986, 1987; Auerbach and Skott, 1988). In dealing with these, we shall assume that Keynesian stability holds.

Now one could argue that the normal rate of capacity utilization is more a norm than a target, and hence that firms may be quite content to run their production capacity at rates of utilization that are within an acceptable range of the normal rate of utilization. If
this is correct, then the analysis pursued so far would still be valid in the long run, as long as the rate of capacity utilization remains within the acceptable range (Dutt, 1990, p. 59).

But less us admit for discussion purposes that this range is very limited. What mechanisms could exist that would bring back the economy towards a normal rate of utilization of capacity, or towards what Sraffians would call fully-adjusted positions (Vianello, 1985)? Two French economists, Duménil and Lévy (1999) have long been arguing that Keynesian economists are mistaken in applying to the long run results arising from the short run. Their claim, in short, is that one should be Kaleckian or Keynesian in the short run, but classical in the long run. What they mean by this is that, in the long run, the economy will be brought back to normal rates of utilization – fully adjusted positions as the Sraffians would say – and that in the long run classical economics will be relevant again. Put briefly, this implies that in the long run a lower propensity to save will drive down the rate of growth of the economy, and that a lower normal profit rate (that is higher real wages and a lower profit share, for a given technology), will also drive down the rate of accumulation. These authors thus reject the paradox of thrift and the paradox of costs, with the latter implying that a reduction in profit margins leads to a higher realized profit rate.

In view of the investment function proposed by Kurz (1990) and by Bhaduri and Marglin (1990), a rejection of the paradox of costs is only incompatible with the canonical Kaleckian model which does not include a profit share or normal profit variable in its investment function. In addition, various authors have shown that the paradox of costs is weakened by the introduction of saving by wage recipients (Blecker, 2002; Lavoie, 1992, p. 344). On the other hand the paradox of thrift is considered to be a robust component of the Kaleckian growth model. Thus one could say that the paradox of thrift is the crucial relationship being at stake here.

Duménil and Lévy (1999) provide a simple mechanism that ought to bring back the economy to normal rates of capacity utilization. They consider that monetary policy is that mechanism. Their model, as shown by Lavoie (2004) and Lavoie and Kriesler (2007), is strongly reminiscent of the New Consensus model, but there is also a great deal of resemblance with Joan Robinson’s inflation barrier and the reaction of the monetary authorities that she describes (1956, p. 238; 1962, p. 60). We can write their model as equations (1A), (2), which we rewrite here for convenience, and equations (3C), (11) and (12):

\[(1A) \quad r = r_n u/u_n \]
\[(2) \quad g_t = s_p r \]
\[(3C) \quad g^t = \gamma + \gamma_u u - \gamma i \]
\[(11) \quad \pi = \chi(u - u_n) \]
\[(12) \quad \Delta i = \epsilon(\pi) \]

where \( i \) is the real rate of interest and \( \pi \) is the rate of inflation. Thus equation (11) is some sort of non-vertical Phillips curve, while equation (12) is a central bank reaction function.\(^6\)

\(^6\) More exactly this exact formulation can be found in the earlier working paper that gave rise to Duménil and Lévy (1999). For a truly New Consensus model, with a vertical Phillips curve, one would need the change in inflation to depend on the discrepancy between the actual and the normal rates of utilization. In
Suppose that this economy is subjected to a Keynesian adjustment mechanism, and that inflation kicks off with a lag. A decrease in the propensity to save will rotate the saving function downwards in Figure 9, bringing the rate of capacity utilization from $u_n$ to $u_1$. Through equation (11), this generates demand inflation, which induces the central bank to raise real interest rates, as shown by equation (12). Interest rates will keep on rising as long as inflation is not brought back to zero. As a consequence, the investment function $g^i$ shifts down gradually. It will stop shifting only when it hits back the normal rate of utilization $u_n$, because this is where inflation is brought back to zero. The end result, however, as can be read off Figure 9, is that the economy now grows at a slower rate, $g_2$ instead of $g_0$.

**INSERT FIGURE 9**

The lesson drawn from this graph is that the economy might be demand-led in the short run, but in the long run it is supply-led. In the long run, the growth rate is determined by the saving function, calculated at the normal rate of capacity utilization, and hence calculated at the normal profit rate: $g^i = s_p r_n$. Thus, a reduction in $s_p$ or $r_n$, in the propensity to save or the normal profit rate, induces a slowdown of the rate of accumulation in the long run. We are back to the dismal science.

**7. The Cambridge price mechanism on its own: cul-de-sac or way-out?**

Are there any alternatives to the return of the dismal science? The old Cambridge story – the one provided by Joan Robinson (1956, 1962) – provides a fully-adjusted position without giving up the paradox of thrift. As is well-known, her suggested investment function is a function of the expected profit rate, itself determined by past realized profit rates, so that, as a simplification we may write:

\[(3D) \quad g^i = \gamma + \gamma r\]

Suppose again that the propensity to save decreases, thus generating the paradox of thrift by bringing the accumulation rate from $g_0$ to $g_1$ while the rate of utilization slides up from $u_n$ to $u_1$, as shown in Figure 10, thus allowing the rate of profit to rise from $r_0$ to $r_1$. Robinson and the Cambridge economists thought however that the economy would be back at its normal rate of utilization in the long run. Their proposed adjustment mechanism is a variant of what we have called the Cambridge price adjustment mechanism (equation (7)), and, recalling equation (1A), it can be written either as equation (13) or equation (13A).

\[(13) \quad \Delta r_n = \varphi(u - u_n) \quad \text{with } \varphi > 0\]

\[(13A) \quad \Delta r_n = \varphi \frac{u_n}{r_n} (r - r_n) \quad \text{with } \varphi > 0\]

---

this case, to avoid a limit cycle, one would need the central bank reaction function to be a function of both the level of inflation (relative to the target inflation rate) and the change in inflation.
With above-normal rates of utilization, profit margins rise. As a result, the profit curve $PC$, as given by equation (1A), rotates down in the lower part of Figure 10, bringing back the actual rate of utilization towards $u_n$. Since the Cambridge investment function depends on the profit rate, it is impervious to the change in the profit margin, so that the growth rate and the profit rate remain at their higher values, $g_1$ and $r_1$. Despite the fully-adjusted position, the paradox of thrift is sustained in the long run. Thus, as pointed out by Marglin (1984, p. 125), in the early Cambridge model, ‘the key assumption is that the rate of capacity utilisation varies on the path between steady-state configurations, but not across steady-growth states’. This means however that there exists a necessary negative relationship (for a given technology) between real wages and accumulation.

**INSERT FIGURE 10**

8. Hysteresis in the long run: Back to Kaleckian results

In a number of places, I have argued that the paradoxes of thrift and costs, as well as the long-run endogeneity of the rate of capacity utilization, could be salvaged even when adopting this kind of Cambridge price adjustment mechanism for the long run (Lavoie, 1992, pp. 417-421; 2003). The reason is that, with bargaining between firms and labour unions, one must distinguish between the normal rate of profit $r_n$, as assessed by firms, and the target rate of return $r_s$ which is incorporated into prices. What the Cambridge price adjustment mechanism of equation (13A) tells us is that the normal rate of profit will change in line with the realized rate of profit. In the long run, these two rates will equate each other, so that $r_n = r$. However, through bargaining and real wage resistance, the target rate of return embodied in the pricing equation will be different from the normal rate of profit as assessed by firms, so that $r_s \neq r_n$ even in the long run. As a consequence, the rate of capacity utilization does not converge to the normal rate of capacity utilization in the long run despite the assumed price adjustment mechanism.

The endogeneity of the actual rate of capacity utilization is thus preserved in both the short and the long run, and the standard Kaleckian results – such as the paradox of thrift, or the paradox of costs if it holds in the assumed configuration – are still vindicated. The above is also consistent with Steindl’s rejection of the intuitive belief that planned excess capacity ought to equal actual long term excess capacity, as he concluded that ‘the degree of utilization actually obtaining in the long run is no safe indication of the planned level of utilization’ (Steindl, 1952, p. 12).

I have taken a different approach in another paper (Lavoie, 1996), introducing two adjustment mechanisms at once instead of a single one, as was done in the previous section that dealt with the dual adjustment process. In that paper, one has to distinguish between fast and slow processes. In the short run, we have a dynamic equilibrium model, based on a pure Keynesian adjustment process. Thus, in the short period aggregate supply

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7 Earlier Cambridge economists such as Robinson, Kaldor and Pasinetti thought that this would occur through some competitive process, whereas Cambridge economists in the 1970s, for instance Alfred Eichner, Wynne Godley, G.C. Harcourt and Adrian Wood thought that this would occur through a price-setting mechanism whereby oligopolistic firms would raise profit margins when trend growth was faster. Kaldor also came to adopt this point of view later in his life.
adjusts quickly enough to aggregate demand for aggregate demand to be at all times equal to aggregate supply. Keynesian stability is thus assumed. But there is also a slow adjustment process that operates in the long run, and that involves two variables. Depending on the exact model being considered, and on the exact adjustment processes being taken into account, various conclusions can be drawn. Cassetti (2006) uses a similar method, but drawing on an adjustment process that involves four variables, including the rate of capital scrapping, which is not considered here.

Price only dynamics

Let us first start with an even simpler Kaleckian model, where investment only depends on the rate of utilization, as sometimes recommended by Dutt (1990, p. 59). We have the following three equations:

\[ r = (r_n/u_n)u \]
\[ g^s = s^p r \]
\[ g^i = \gamma + \gamma u_u \text{ with } \gamma > 0 \]

accompanied by the following two long-run adjustment processes, which, with the present model, obviously only have an impact on the pricing and saving equations:

\[ \Delta r_n = \phi(r^* - r_n) = \phi(r_n/u_n)(u^* - u_n) \]
\[ \Delta u_n = \sigma(u^* - u_n) \]

where \( u^* \) and \( r^* \) are the medium run values of the model.

With equation (3E) we set aside for the moment the complications that could arise from considering the shape of the investment function. Whereas I presume that most of my colleagues would accept the notion that the normal rate of profit would be influenced by past realized profit rates, as suggested in equation (13A), there is a certain number of authors, such as Skott (2008), who are rather reluctant to accept the argument that the normal rate of capacity utilization will also be influenced by past realized rates of utilization, as proposed in equation (15). While I have some sympathy for their objections, having myself argued that the normal rate of capacity utilization may be more influenced by the past variance of actual rates of utilization than by their past realized values (Lavoie, 1992, p. 330), there is nevertheless some evidence that normal rates of utilization are influenced by past realized values. For instance, Clifton (1983, p. 26) remarks that cost-plus prices are based on standard volumes of utilization taken from historical data that cover several business cycles. In addition, Joan Robinson has herself argued that normal rates of profit and of capacity utilization were subjected to adaptive adjustment processes, as the following quote shows:

Where fluctuations in output are expected and regarded as normal, the subjective-normal price may be calculated upon the basis of an average or standard rate of output, rather than capacity.... Profits may exceed of fall short of the level on the basis of which the subjective-normal prices were conceived. Then experience gradually modifies the views of
entrepreneurs about what level of profit is obtainable, or what the average utilization of plant is likely to be over its lifetime, and so reacts upon subjective-normal prices for the future (Robinson, 1956, pp. 186, 190).

Looking now at equations (14) and (15), we see that what we have is a model which is a particular case of the dual adjustment mechanism that we described earlier and that gave rise to Table 1. Keynesian stability is assumed, and since $\gamma_r = 0$, the slope of $dm/du$ is necessarily negative, implying a wage-led model. The relative size of the adjustments to the normal profit rate and the normal utilization rate explain whether the model is driven by a Cambridge price adjustment process or by a Radical price adjustment process. Thus, this model corresponds to entries A and B in Table 1.

The model reaches its long-run equilibrium – its fully adjusted position – when $\Delta u = \Delta r = 0$, that is when $u^* - u_n = 0$, and using equation (4A) with $\gamma_r = 0$, we can compute that this will occur when:

\[
(r_n^{**} = (\gamma_u u_n^{**} + \gamma)/s_p)
\]

Figure 11 illustrates this slow adjustment process that occurs in the long run. The economy is initially in a fully adjusted position at $u_n^{**}$ and $r_n^{**}$ on the demarcation line. Then there is a decrease in the propensity to save, which shifts up the demarcation line, raising both the short-run actual rate of profit and rate of capacity utilization. The other upward sloping line, marked as $r_n = (m/v)u_n$, represents the relationship between the normal rate of profit and the normal rate of utilization when profit margins don’t change. With the slow adjusting mechanism associated with normal values, the economy will move to point A, corresponding to entry A in Table 1, if the normal profit rate rises faster than the normal rate of utilization (that is if $\phi > \tau$). In this case, as shown in the figure, profit margins are rising, and this corresponds to a kind of Cambridge price adjustment mechanism. If profit margins remain constant while normal rates of profit and of capacity utilization rise, then the economy gets to point M (if $\phi = \tau$). Finally, if the normal profit rate rises more slowly than the normal rate of utilization (if $\phi < \tau$) then the economy will move to point BS in the stable case, while it will move along the B1 arrowhead in the unstable case. Instability will occur if the slope of the trajectory towards the new fully adjusted position is less steep than the slope of the new demarcation line, given by equation (13), that is if:

\[
dr_n/du_n = (\phi/\tau)(r_n/u_n) < \gamma_u/s_p \text{ or if } (\phi/\tau) < (\gamma_u u_n)/(s_p r_n).
\]

INSERT FIGURE 11

An interesting characteristic of the present model is that it features what Setterfield (1993) calls deep endogeneity. The new fully adjusted position depends on the previous fully adjusted position. Very clearly, it also depends on the reaction parameters during the transition or traverse process, and hence we may also say that it is path-dependent. It retains the main properties of the canonical Kaleckian growth model, as shown here with the paradox of thrift.

**Combining price and investment dynamics**
We now examine another variant of the Kaleckian model, by assuming that entrepreneurs entertain the same value of the normal rate of capacity utilization, both in the pricing equation and in the investment equation. To take that into consideration, we must yet modify again the investment equation, adopting an equation which is often found in the literature. With equations (1A), (2), (11) and (12), we have:

\[(3F) \quad g^i = \gamma + \gamma u(u - u_n) \quad \text{with } \gamma > 0\]

While such a model would seem to be more complicated than the previous one, in fact it is the opposite. What happens is that the fully-adjusted position gets simplified, thanks to equation (3F), because \(\Delta u = \Delta r = 0\), when \(u^* - u_n = 0\), which means that \(g^{**} = \gamma\) in the fully-adjusted position, and hence, using equation (2), it implies that:

\[(17) \quad r_n^{**} = \gamma / s_p\]

Once more we can illustrate the slow long-run adjustment process, with Figure 12, which is a degenerate version of Figure 11. The demarcation line is now a simple horizontal line, given by equation (17), which shifts up when the propensity to save is lower (or animal spirits, as proxied by \(\gamma\), are higher). Both the normal profit rate and the normal rate of utilization rise under such a change. The model is unconditionally stable. But we get a degenerate version of the Kaleckian model. The growth rate of the economy is stuck at \(\gamma\) in the long run, so there is no paradox of thrift any more in fully-adjusted positions, although lower propensities to save will generate higher normal rates of profit and higher normal rates of capacity utilization.

**INSERT FIGURE 12**

**Investment dynamics and hysteresis**

Finally, one may wish to focus on the long-run dynamics involving only the investment function, as does also Dutt (1997, pp. 245-8). In this case, we consider once again investment function (3F), along with equations (1) and (2):

\[(1) \quad r = mu/v\]
\[(2) \quad g^s = s_p r\]
\[(3F) \quad g^i = \gamma + \gamma u(u - u_n) \quad \text{with } \gamma > 0\]

The \(\gamma\) parameter in investment function (3F) is often interpreted as the secular growth rate of the economy, or the expected growth rate of sales. Firms speed up accumulation, relative to this secular growth rate, when current capacity utilization exceeds the target, thus trying to catch up. One would also think that the expected trend growth rate is influenced by past values of the actual growth rate. With normal rates of capacity utilization also being influenced by past actual rates, the two dynamic equations are given by:
Making the proper substitutions, these two equations get rewritten as:

\[
(15A) \quad \Delta u_n = \frac{\sigma (\gamma - \alpha u_n)}{\alpha - \gamma_u} \\
(18A) \quad \Delta \gamma = \frac{\Omega \gamma_u (\gamma - \alpha u_n)}{\alpha - \gamma_u}
\]

with \( \alpha = \frac{s_p m}{v} \), and hence the differential function relevant to the perceived growth trend is:

\[
(18B) \quad \Delta \gamma = \frac{\Omega \gamma_u}{\sigma} \Delta u
\]

**INSERT FIGURE 13**

Thus once again we have a continuum of equilibria, such that \( \Delta u_n = \Delta \gamma = 0 \), shown in Figure 13, and which corresponds to the \( \gamma = au_n = (s_p m/v)u_n \) linear equation. With a decrease in the propensity to save, the continuum of long-run equilibria rotates downward, and two cases arise. Either the dynamic equations (15) and (18) describe a stabilizing process, in which case the normal rate of utilization and the perceived growth trend rise up to a point such as \( A_S \) in Figure 13, or the process is unstable, as shown by arrowhead \( A_U \). The process will be stable provided the transitional path has a smaller slope than that of the new demarcation line, that is provided we have \( d\gamma/du_n = \Omega \gamma_u/\sigma < \alpha \), which means that \( s_p m/v > (\Omega/\sigma)\gamma_u \). If the Keynesian stability condition holds, that is if \( s_p m/v > \gamma_u \), then a sufficient condition for dynamic stability is simply \( \sigma > \Omega \). In other words, the Harrodian instability effect, represented by equation (18) which tells us that entrepreneurs will raise their expectations about future growth rates whenever current realized growth rates exceed the current trend estimate, must not be too large.

Other mechanisms have recently been proposed to tame Harrodian instability or to bring the Kaleckian model back to normal rates of capacity utilization (Shaikh, 2007), but the discussion of these mechanisms would overly extend the present chapter.

**9. Conclusion**

The Kaleckian growth model has proven to be highly flexible and fruitful, being able to incorporate the concerns of several different schools of thought. I have not dealt with the important question of the discrepancy between the rate of accumulation as determined by the Kaleckian model and the natural rate of growth. Neither have I dealt with finance, debt, and stock-flow issues (Taylor, 2004, pp. 272-8). But all these questions are discussed elsewhere in this book (Dutt 2009; Hein and Trick, 2009).
References


Slide 4

Figure 4

Slide 5

Figure 5

Slide 6

Figure 6
Slide 10

Slide 11

Slide 12
\[ \Delta u = 0 \]
\[ \Delta \gamma = 0 \]