

POSSIBLE RESEARCH PROJECTS FOR GRADUATE STUDENTS

I have 2 main lines of research at the moment. First, I am looking at applications of logic and category theory to physics. It turns out that certain ideas in quantum mechanics can be expressed more abstractly just using category theory. The most important structure in all of quantum mechanics is the tensor product of Hilbert spaces, which allows one to explain entanglement of particles. Entanglement is at the root of things like the Einstein-Podolsky-Rosen paradox. It is the basis for processes like quantum teleportation. But there is a form of logic called linear logic which allows one to use logical analysis to consider such tensor products. Basically in linear logic, you treat the tensor product like a logical conjunction. Then quantum processes can be examined by assigning them logical deductions in a linear logic deductive system. I have been working with the Quantum Group at the Oxford Computing Lab to extend these ideas to take into account the interaction of quantum mechanics and gravity.

My grad student Marc Comeau and I are working to extend the above ideas to take into account the structure of spacetime. Any protocol such as quantum teleportation takes place in spacetime, and so its effects must be accounted for. The most convenient mathematical structure for doing so is *algebraic quantum field theory*. AQFT is also an exciting mathematical structure, which combines ideas from category theory and functional analysis.

The other application of logic and category theory that I am interested in is to differential geometry. There is a notion of logic called differential linear logic which can be used to bring logical techniques to analyze smooth (infinitely differentiable) functions. Basically linear logic is extended to include differentiation, which is treated as a logical inference rule. We have worked out the basic theory, and are now looking for interesting models. We are using ideas from functional analysis and algebraic geometry and many other areas.

There are all sorts of projects for graduate students to work on in these two fields.