Since the left side of this equality is a linear function, we try $f(x) = ax + b$ where a and b are constants. But $f(x) = ax + b$ implies that $f'(x) = a$. Thus we must have

$$2x = 5(ax + b) + a = 5ax + (a + 5b)$$

This will be an identity if, and only if,

$$5a = 2 \text{ and } a + 5b = 0$$

Solving the system

$$\begin{cases} 5a = 2 \\ a + 5b = 0 \end{cases}$$

we get

$$a = \frac{2}{5} \text{ and } b = \frac{-2}{25}$$

Thus

$$\int e^{5x} 2x\, dx = e^{5x}\left(\frac{2}{5}x - \frac{2}{25}\right) + C$$

Check:
$$D_x\left[e^{5x}\left(\frac{2}{5}x - \frac{2}{25}\right) + C\right] = e^{5x}\frac{2}{5} + \left(\frac{2}{5}x - \frac{2}{25}\right)e^{5x}(5) + 0$$
$$= e^{5x}\left[\frac{2}{5} + 5\left(\frac{2}{5}x - \frac{2}{25}\right)\right]$$
$$= e^{5x}\left[\frac{2}{5} + 2x - \frac{2}{5}\right] = e^{5x}2x$$

The answer is correct.

Do Exercise 23. (See Exercises 45 and 46.) ■

Exercise Set 4.2

In Exercises 1–24, evaluate the given integral using a u-substitution and check your answer by differentiation.

1. $\int (x^3 + 5)^{10} x^2\, dx$

2. $\int \sqrt[4]{x^3 + 8}\, x^2\, dx$

3. $\int \frac{x^3}{(x^4 + 3)^5}\, dx$

4. $\int \frac{x^4}{\sqrt{x^5 + 6}}\, dx$

5. $\int (x^2 + 2x + 5)^{10}(x + 1)\, dx$

6. $\int (x^4 + 8x + 6)^6\, 3(x^3 + 2)\, dx$

7. $\int \sqrt[3]{x^2 + 6x + 1}(5x + 15)\, dx$

8. $\int \frac{2x^2 + 8x + 6}{\sqrt[5]{x^3 + 6x^2 + 9x + 1}}\, dx$

9. $\int \frac{7x^4}{x^5 + 4x}\, dx$

10. $\int \frac{x + 2}{x^2 + 4x + 5}\, dx$

11. $\int \frac{e^{2x}}{e^{2x} + 1}\, dx$ (*Hint:* Let $u = e^{2x} + 1$.)

12. $\int \frac{1}{x \ln|x|}\, dx$ (*Hint:* Let $u = \ln|x|$.)

13. $\int (e^{5x} + 1)^{12} 2e^{5x}\, dx$

14. $\int \frac{5e^{3x}}{\sqrt{e^{3x} + 2}}\, dx$

15. $\int e^{6x}\, dx$

16. $\int e^{x^5} 3x^4\, dx$

17. $\int e^{x^3 + 6x + 10}(x^2 + 2)\, dx$

r function, we try $f(x) = ax + b$ where a
plies that $f'(x) = a$. Thus we must have

$b)$

$+ \left(\frac{2}{5}x - \frac{2}{25}\right)e^{5x}(5) + 0$

$\frac{2}{5} + 5\left(\frac{2}{5}x - \frac{2}{25}\right)\Big]$

$\frac{2}{5} + 2x - \frac{2}{5}\Big] = e^{5x}2x$

46.) ■

$\frac{}{x}\, dx$

$\frac{2}{x+5}\, dx$

$\frac{}{1}\, dx$ (*Hint:* Let $u = e^{2x} + 1$.)

dx (*Hint:* Let $u = \ln|x|$.)

$1)^{12}2e^{5x}\, dx$

14. $\int \frac{5e^{3x}}{\sqrt{e^{3x}+2}}\, dx$

16. $\int e^{x^5}3x^4\, dx$

18. $\int e^{x^3+6x^2+15x+4}(2x^2 + 8x + 10)\, dx$

19. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$

20. $\int \frac{e^{2x}}{(e^{2x}+1)\ln(e^{2x}+1)}\, dx$ (*Hint:* Let $u = \ln(e^{2x} + 1)$.)

21. $\int 2^{x^2}x\, dx$

22. $\int 5^{x^2+4x+3}(x + 2)\, dx$

23. $\int 5xe^{3x}\, dx$ (*Hint:* Try $(ax + b)e^{3x}$ where a and b are constants. See Example 4.)

24. $\int x^2e^{5x}\, dx$ (*Hint:* Try $(ax^2 + bx + c)e^{5x}$ where a, b, and c are constants. See Example 4.)

In Exercises 25–29, the velocity function of a particle moving along a directed line is given. The value of the position function at a certain instant is also given. Derive the formula for the position function. Time is measured in seconds and distance in centimeters.

25. $v(t) = (t^2 + 1)^{1/2}3t,\ s(0) = 10$

26. $v(t) = (t^3 + 6t + 5)^{1/3}(t^2 + 2),\ s(1) = 5$

27. $v(t) = e^{-t^2}3t,\ s(0) = 4$

28. $v(t) = \frac{3t^3}{t^4 + 5},\ s(0) = 8$

29. $v(t) = 5^{t^2+1}(4t),\ s(0) = 25$

In Exercises 30–33, the marginal cost function at a production level of q units of a commodity is given. In each case the fixed cost is also given. Find the cost function.

30. $M_c(q) = \frac{2q^3 + 1}{\sqrt[5]{(q^4 + 2q + 3)^4}}$, \$50,000

31. $M_c(q) = \frac{q + 3}{\sqrt[3]{(q^2 + 6q + 1)^2}}$, \$30,000

32. $M_c(q) = \frac{e^{0.001q}}{(e^{0.001q} + 5)^{11/10}}$, \$10,000

33. $M_c(x) = \frac{2q + 1}{q^2 + q + 5}$, \$40,000

34. Verify Formula 1 by differentiating the right side.

35. Verify Formula 2. **36.** Verify Formula 3.

37. Verify Formula 4. **38.** Verify Formula 5.

39. Verify Formula 6. **40.** Verify Formula 7.

In Exercises 41–44, the acceleration function of a particle moving along a directed line is given. The values of the velocity and position functions at a certain instant are also given. Derive the formula for the position function. Time is measured in seconds and distance in feet.

41. $a(t) = e^{4t};\ v(0) = 12,\ s(0) = 25$

42. $a(t) = e^{t/3};\ v(0) = 6,\ s(0) = -35$

43. $a(t) = te^{3t};\ v(0) = 8,\ s(0) = 40$ (*Hint:* See Example 4 and Exercises 23, 24.)

44. $a(t) = 5t^2e^{5t};\ v(0) = 45,\ s(0) = 20$ (*Hint:* See Example 4 and Exercises 23, 24.)

45. A student attempted to find the integral of Example 4 as follows. He let $u = 5x$, $du = 5\, dx$, $dx = \frac{du}{5}$ and wrote

$$\int e^{5x}2x\, dx = 2x\int e^{5x}\, dx = 2x\int e^{u}\frac{du}{5}$$
$$= \frac{2x}{5}\int e^u\, du = \frac{2x}{5}e^u + C = \frac{2x}{5}e^{5x} + C$$

Show that this answer is wrong. (*Hint:* Differentiate the function he obtained.)

46. A student attempted to find the integral of Example 4 as follows. She guessed that the answer must be of the form ke^{5x} and wrote

$$D_xke^{5x} = ke^{5x}(5) = 5ke^{5x} = e^{5x}2x$$

She solved for k, obtained $k = \frac{2x}{5}$, and concluded that

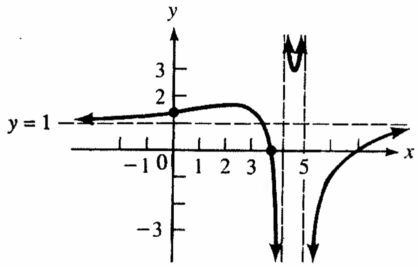
$$\int e^{5x}2x\, dx = \frac{2x}{5}e^{5x} + C$$

Show that her answer is incorrect and explain where she made her error.

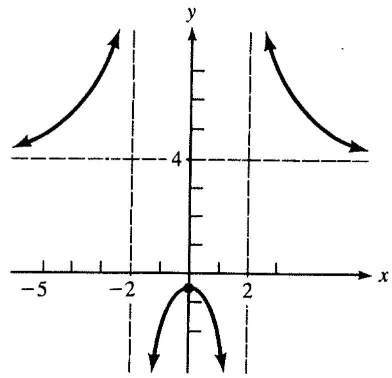
4.3 Integration by Parts

In the preceding two sections, we used differentiation formulas to obtain integration formulas. In this section, we introduce a new method based on the product rule of

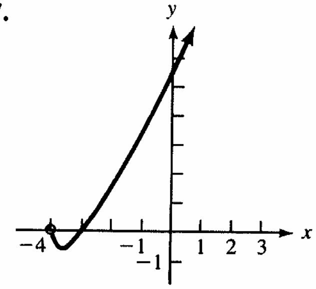
23. The line $y = 1$ is a horizontal asymptote to the left and right. The lines $x = 5$ and $x = \frac{13}{3}$ are vertical asymptotes. The x-intercepts are 4 and 7. The y-intercept is $\frac{84}{65}$.



25. The line $y = 4$ is a horizontal asymptote to the left and right. The lines $x = 2$ and $x = -2$ are vertical asymptotes. $-\frac{1}{2}$ is the y-intercept and there are no x-intercepts.



27.



29. $q^3 - 36q^2 + 432q,\ 0 \le q \le 18,\ 18{,}000$
31. \$7

Exercise Set 4.1 (Pages 311–312) Antidifferentiation

13. $\frac{1}{2}x^4 + 5x + c$ **15.** $\frac{1}{4}\ln|t^4 + 1| + c$ **17.** $\frac{3}{204}(x^4 + 1)^{51} + c$ **19.** $\frac{7}{4}e^{s^4} + c$ **21.** $\frac{1}{3}\ln|x^3 + 3x + 1| + c$
23. $\frac{4}{7}t^{7/2} + 3t^2 + t + c$ **25.** $\frac{3}{20}(x^2 + 1)^{10/3} + c$ **27.** $\frac{5}{4}\ln|x^4 + 1| + c$ **29.** $.5e^{t^2+2t+5} + c$ **31.** $16t^2 + 15t + 20$
33. $2t^3 + 2t^2 + t - 10$ **35.** $\frac{1}{3}\ln|t^3 + 3t + 1| + 50$ **37.** $-6t^2 + 3t + 60,\ -2t^3 + 1.5t^2 + 60t - 114.5$
39. $\frac{100}{3}e^{.03t} - \frac{85}{3},\ \frac{10{,}000}{9}e^{.03t} - \frac{85}{3}t - \frac{9100}{9}$ **41.** $\frac{.01}{3}q^3 - 3q^2 + 15q + 45{,}000$ **43.** $-1500e^{-.002q} + 36{,}500$
45. $g(t) = 5t^{3/2} + 6$

Exercise Set 4.2 (Pages 318–319) The U-Substitution

1. $\frac{1}{33}(x^3 + 5)^{11} + c$ **3.** $\dfrac{-(x^4 + 3)^{-4}}{16} + c$ **5.** $\dfrac{(x^2 + 2x + 5)^{11}}{22} + c$ **7.** $\frac{15}{8}(x^2 + 6x + 1)^{4/3} + c$
9. $\frac{7}{5}\ln|5x + 4| + c$ **11.** $\frac{1}{2}\ln|e^{2x} + 1| + c$ **13.** $\frac{2}{65}(e^{5x} + 1)^{13} + c$ **15.** $\frac{1}{6}e^{6x} + c$ **17.** $\frac{1}{3}e^{x^3+6x+10} + c$
19. $2e^{\sqrt{x}} + c$ **21.** $\frac{1}{2\ln 2}2^{x^2} + c$ **23.** $\left(\frac{5}{3}x - \frac{5}{9}\right)e^{3x} + c$ **25.** $(t^2 + 1)^{3/2} + 9$ **27.** $\frac{-3}{2}e^{-t^2} + \frac{11}{2}$
29. $\frac{2}{\ln 5}5^{t^2+1} + 25 - \frac{10}{\ln 5}$ **31.** $\frac{3}{2}\sqrt[3]{q^2 + 6q + 1} + 29998.5$ **33.** $\ln|q^2 + q + 5| + 40{,}000 - \ln 5$ **41.** $\frac{1}{16}e^{4t} + \frac{47}{4}t + \frac{399}{16}$
43. $\frac{1}{9}te^{3t} - \frac{2}{27}e^{3t} + \frac{73t}{9} + \frac{1082}{27}$

Exercise Set 4.3 (Pages 324–325) Integration by Parts

1. $\frac{3}{8}xe^{8x} - \frac{3}{64}e^{8x} + c$ **3.** $\frac{1}{2}(5x + 6)e^{2x} - \frac{5}{4}e^{2x} + c$ **5.** $\frac{1}{4}(3x + 1)e^{4x} - \frac{3}{16}e^{4x} + c$ **7.** $\frac{1}{3}x^3e^{x^3} - \frac{1}{3}e^{x^3} + c$
9. $x\ln|x^3| - 3x + c$ **11.** $x^2\ln|x + 3| - \dfrac{x^2}{2} + 3x - 9\ln|x + 3| + c$ **13.** $\dfrac{-1}{2x^2}\ln|x| - \dfrac{1}{4x^2} + c$
15. $2\sqrt{x}\ln|x| - 4\sqrt{x} + c$ **17.** $\dfrac{x}{\ln 3}(3^x) - \dfrac{1}{(\ln 3)^2}\cdot 3^x + c$
19. $\frac{1}{4}x^4\ln|x + 4| - \frac{1}{16}x^4 + \frac{1}{3}x^3 + 2x^2 - 16x + 64\ln|x + 4| + c$ **21.** $\frac{3x}{4}(x + 2)^{4/3} - \frac{9}{28}(x + 2)^{7/3} + c$