MATH 1300-MIDTERM-2004

NAME and I.D.# Solutions, Version 1

Instructions– This exam consists of 6 multiple choice questions and 2 long answer questions. The multiple choice questions are worth 6 points each, and the long answer questions are as indicated. The total value of the exam is 60 points.

Place your answers to the multiple choice questions in the boxes below. All your work on the long answer questions must be clearly marked. You may use the backs of pages. **If you need additional scrap paper, it will be provided by the proctors.**

NO CALCULATORS. NO BOOKS. NO NOTES.

On the long answer questions, you must show your work.

Answers:

#1	#2	#3	#4
#5	#6		

Multiple Choice Section-Question 1- Solve the following inequality:

$$2x^2 + 4 < 2 + 5x$$

A) $(\frac{1}{3}, 2)$ B) $(\frac{1}{2}, 4)$ C) $(\frac{1}{2}, 2)$ D) $(\frac{1}{4}, 4)$ E) $(1, 4)$

$2x^2 + 4 < 2 + 5x$

$2x^2 - 5x + 2 < 0$

$(2x-1)(x-2) < 0$

2 cases either

① $2x - 1 < 0$ and $x - 2 > 0$

i.e. $2x < 1$ and $x > 2$

So $x < \frac{1}{2}$ and $x > 2$

Impossible

or $2x - 1 > 0$ and $x - 2 < 0$

or $x > \frac{1}{2}$ and $x < 2$

so $x \in (\frac{1}{2}, 2)$

Question 2- Consider the following function:

$$f(x) = \begin{cases} x^2 - x - 3 & \text{if } x > 3 \\ ax + 2 & \text{otherwise} \end{cases}$$

What value must the constant a be for the function to be continuous at $x = 3$?

A) $\frac{1}{3}$ B) $\frac{1}{4}$ C) $\frac{-1}{4}$ D) $\frac{2}{3}$ E) $\frac{-1}{2}$

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (x^2 - x - 3) = 3$

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} ax + 2 = 3a + 2$

So we must have $3a + 2 = 3$ or $a = \frac{1}{3}$

Question 3- Find the equation of the tangent line of the function $f(x) = \frac{9}{x^2+x+3}$ at $x = 2$.

A) $y = \frac{x}{8} + \frac{7}{8}$ B) $y = \frac{-5x}{8} + \frac{9}{4}$ C) $y = -x + \frac{5}{3}$

D) $y = \frac{-5x}{9} + \frac{19}{9}$ E) $y = 2x - \frac{4}{3}$

$$f'(x) = \frac{-9 \cdot (2x+1)}{(x^2+x+3)} \quad \text{So} \quad f'(2) = \frac{-9(5)}{9^2} = \frac{-5}{9}.$$

When $x=2$, $f(x)=1$. So $1 = \frac{-5}{9}(2)+b$ or $b = \frac{19}{9}$

So $\boxed{y = \frac{-5x}{9} + \frac{19}{9}}$

Question 4-Calculate:

$$\lim_{x\to 1} \frac{x^2+3x-4}{x^2-1}$$

A) $\frac{3}{2}$ B) $\frac{5}{2}$ C) $\frac{4}{3}$ D) $\frac{3}{4}$ E) $\frac{3}{5}$

$$\frac{x^2+3x-4}{x^2-1} = \frac{(x+4)(x-1)}{(x+1)(x-1)} = \frac{x+4}{x+1}, \text{ when } x \neq 1$$

So $\lim_{x\to 1} \frac{x+4}{x+1} = \frac{5}{2}$

Question 5 Solve

$$5^{2x} = e^4$$

A) $x = \frac{2\ln(5)}{4}$ B) $x = \frac{2\ln(4)}{5}$ C) $x = \frac{4}{\ln(25)}$ D) $x = \frac{\ln(4)}{5}$ E) $x = \frac{\ln(40)}{4}$

$$5^{2x} = e^4$$

$$\ln(5^{2x}) = \ln(e^4) = 4$$

$$2x\ln(5) = 4$$

$$x = \frac{4}{2\ln(5)} = \frac{4}{\ln(5^2)} = \frac{4}{\ln(25)}$$

Question 6 Let $f(x) = x\sqrt{2x-2}$. Find $f'(x)$ at $x = 3$.

A) $\frac{5}{2}$ B) $\frac{2}{3}$ C) $\frac{-3}{2}$ D) $\frac{-6}{19}$ E) $\frac{7}{2}$

$$f(x) = x(2x-2)^{1/2}$$

$$f'(x) = (2x-2)^{1/2} + x\left[\left(\frac{1}{2}\right)(2x-2)^{-1/2}(2)\right]$$

$$= \sqrt{2x-2} + \frac{x}{\sqrt{2x-2}}$$

$$f'(3) = \sqrt{4} + \frac{3}{\sqrt{4}} = 2 + \frac{3}{2} = \frac{7}{2}$$

Long Answer Questions-Question 1 (12 points)

One thousand dollars is invested at a rate of 6%, compounded continuously.

- How much will be in the account in 5 years?
- How long is required for the initial investment to triple?

$$y = P_0 e^{rt} = 1{,}000\, e^{(.06)t} = A(t)$$

Part 1

$$A(5) = 1{,}000\, e^{(.06)(5)} = 1{,}000\, e^{.30}$$

Part 2

We must solve the equation

$$3{,}000 = 1{,}000\, e^{.06t} \quad \text{for } t$$

$$3 = e^{.06t}$$

$$\ln(3) = \ln\left(e^{.06t}\right) = .06t$$

$$\text{So} \quad t = \frac{\ln(3)}{.06} = \frac{50\ln(3)}{3}$$

Question 2 (12 points)

Find the equation to the tangent line to the graph of $x^4y^2 = 1$ at the point $(\frac{1}{2}, 4)$

Using Implicit Differentiation:

$$x^4(2yy') + 4x^3y^2 = 0$$

Now plug in point

$$\left(\tfrac{1}{2}\right)^4(2)(4)y' + 4\left(\tfrac{1}{2}\right)^3(4)^2 = 0$$

$$\tfrac{1}{2}y' + 8 = 0$$

or $y' = -16$

So line equation is $y = -16x + b$

A point on the line is $(1/2, 4)$. So plug in to solve for b.

$$4 = -16(1/2) + b$$

or $b = 12$

So $\boxed{y = -16x + 12}$

MATH 1301-MIDTERM-2004

NAME and I.D.# Solutions, Version 2

INSTRUCTIONS– This exam consists of 6 multiple choice questions and 2 long answer questions. The multiple choice questions are worth 6 points each, and the long answer questions are as indicated. The total value of the exam is 60 points.

Place your answers to the multiple choice questions in the boxes below. All your work on the long answer questions must be clearly marked. You may use the backs of pages. **If you need additional scrap paper, it will be provided by the proctors.**

NO CALCULATORS. NO BOOKS. NO NOTES.

On the long answer questions, you must show your work.

Answers:

#1	#2	#3	#4
#5	#6		

Multiple Choice Section-Question 1- Solve the following inequality:

$$2x^2 + 1 < 9x - 3$$

A) $(\frac{1}{3}, 2)$ B) $(\frac{1}{2}, 4)$ C) $(\frac{1}{2}, 2)$ D) $(\frac{1}{4}, 4)$ E) $(1, 4)$

$2x^2 - 9x + 4 < 0$

$(2x-1)(x-4) < 0$

2 cases:

Case 1) $2x - 1 < 0$ and $x - 4 > 0$

So $x < 1/2$ and $x > 4$

IMPOSSIBLE

Case 2

$2x - 1 > 0$ and $x - 4 < 0$

So $x > 1/2$ and $x < 4$

So $x \in (\frac{1}{2}, 4)$

Question 2- Consider the following function:

$$f(x) = \begin{cases} x^2 - x + 1 & \text{if } x > 2 \\ ax + 4 & \text{otherwise} \end{cases}$$

What value must the constant a be for the function to be continuous at $x = 2$?

A) $\frac{1}{3}$ B) $\frac{1}{4}$ C) $\frac{-1}{4}$ D) $\frac{2}{3}$ E) $\frac{-1}{2}$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} x^2 - x + 1 = 3$

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} ax + 4 = 2a + 4.$

So $3 = 2a + 4$

$a = -1/2$

Question 3- Find the equation of the tangent line of the function $f(x) = \frac{8}{x^2+x+2}$ at $x = 2$.

A) $y = \frac{x}{8} + \frac{7}{8}$ B) $y = \frac{-5x}{8} + \frac{9}{4}$ C) $y = -x + \frac{5}{3}$

D) $y = \frac{2x}{3} - \frac{2}{3}$ E) $y = 2x - \frac{4}{3}$

$f(x) = 8(x^2+x+2)^{-1}$

$f'(x) = -8(x^2+x+2)^{-2}(2x+1)$

$= \frac{-8}{(x^2+x+2)^2}(2x+1)$

$f'(2) = \frac{-8}{64} \cdot (5) = -\frac{5}{8}$

when $x=2$, $f(x)=1$.

So

$y = -\frac{5}{8}x + b$

$1 = -\frac{5}{8}(2) + b$

$b = \frac{18}{8} = \frac{9}{4}$

$\boxed{y = -\frac{5}{8}x + \frac{9}{4}}$

Question 4-Calculate:

$$\lim_{x \to 1} \frac{x^2+x-2}{x^2-1}$$

A) $\frac{3}{2}$ B) $\frac{5}{2}$ C) $\frac{4}{3}$ D) $\frac{3}{4}$ E) $\frac{3}{5}$

$\frac{x^2+x-2}{x^2-1} = \frac{(x+2)(x-1)}{(x+1)(x-1)} = \frac{x+2}{x+1}$, when $x \neq 1$

So $\lim_{x\to 1} \frac{x^2+x-2}{x^2-1} = \lim_{x \to 1} \frac{x+2}{x+1} = \frac{3}{2}$

Question 5 Solve

$$5^{2x} = e^3$$

A) $x = \frac{2\ln(5)}{3}$ B) $x = \frac{3\ln(3)}{5}$ C) $x = \frac{\ln(3)}{5}$ D) $x = \frac{3}{\ln(25)}$ E) $x = \frac{\ln(40)}{4}$

$$\ln(5^{2x}) = \ln(e^3) = 3$$

$$2x\ln(5) = 3$$

$$x = \frac{3}{2\ln(5)} = \frac{3}{\ln(25)}$$

Question 6 Let $f(x) = x\sqrt{2x+1}$. Find $f'(x)$ at $x = 4$.

A) $\frac{13}{3}$ B) $\frac{2}{3}$ C) $\frac{-3}{8}$ D) $\frac{-6}{19}$ E) $\frac{4}{3}$

$$f'(x) = \sqrt{2x+1} + x\left[\left(\frac{1}{2}\right)(2x+1)^{-1/2}(2)\right]$$

$$= \sqrt{2x+1} + \frac{x}{\sqrt{2x+1}}$$

$$f'(4) = \sqrt{9} + \frac{4}{\sqrt{9}} = 3 + \frac{4}{3} = \frac{13}{3}$$

Long Answer Questions-Question 1 (12 points)
One thousand dollars is invested at a rate of 5%, compounded continuously.

- How much will be in the account in 6 years?
- How long is required for the initial investment to double?

Part A) $P(t) = P_0 e^{rt}$, since compounding is continuous.

In this case $P(t) = 1{,}000\, e^{.05t}$

So $P(6) = 1{,}000\, e^{.05(6)} = 1{,}000\, e^{.3}$

Part B) We must solve for t:

$$2{,}000 = 1{,}000\, e^{.05t}$$

$$2 = e^{.05t}$$

$$\ln(2) = .05t$$

$$t = \frac{\ln(2)}{.05} = 20 \ln(2)$$

Question 2 (12 points)

Find the equation to the tangent line to the graph of $x^2y^4 = 1$ at the point $(4, \frac{1}{2})$

Solve by implicit differentiation:

$$2xy^4 + x^2(4y^3y') = 0$$

Plug in $(4, 1/2)$

$$2(4)(1/2)^4 + 4^2(4)(1/2)^3 y' = 0$$

$$\frac{1}{2} + 8y' = 0$$

$$y' = -1/16$$

So $y = mx + b = -\frac{1}{16}x + b$ Plug in $(4, 1/2)$

$$\frac{1}{2} = -\frac{1}{16}(4) + b$$

$b = \frac{3}{4}$. So $\boxed{y = -\frac{1}{16}x + 3/4}$