MATH 1300-MIDTERM 2-2004

SOLUTIONS - Version 1

NAME and I.D.#_____

Instructions- This exam consists of 6 multiple choice questions and 2 long answer questions. The multiple choice questions are worth 6 points each, and the long answer questions are as indicated. The total value of the exam is 60 points.

Place your answers to the multiple choice questions in the boxes below. All your work on the long answer questions must be clearly marked. You may use the backs of pages. If you need additional scrap paper, it will be provided by the proctors.

NO CALCULATORS. NO BOOKS. NO NOTES.

On the long answer questions, you must show your work.

Answers:

#1	#2	#3	#4

#5	#6

Multiple Choice Section-Question 1- If $g(x) = \ln(4x^2 - 3)$, what is $g'(1)$?

A) 8 B) 7 C) 6 D) 5 E) 4

$g'(x) = \frac{1}{4x^2-3}(8x)$, by chain rule:

So $g'(1) = \frac{1}{4(1)^2-3} \cdot 8 = 8$

Question 2- A certain bacteria grows exponentially. The initial population was 400,000. One hour after the experiment started, the population had grown to 600,000. What was the population two hours after the experiment started.

A) 500,000 B) 650,000 C) 600,000 D) 800,000 E) 900,000

$P(t) = P_0 b^t = 400{,}000\, b^t$

$P(1) = 600{,}000$. So $600{,}000 = 400{,}000\, b$

or $b = \frac{3}{2}$

$P(t) = 400{,}000\left(\frac{3}{2}\right)^t$. So $P(2) = 400{,}000\left(\frac{9}{4}\right)$

$= 900{,}000$

Question 3- Let $f(x) = x^3 - 3x^2 + 3$. On what interval is the function decreasing?

A) $(1, \infty)$ B) $(1, 3)$ C) $(2, 3)$ D) $(-1, 2)$ E) $(0, 2)$

$$f'(x) = 3x^2 - 6x = 3x(x-2)$$

So there are CP's at $x=0$ and $x=2$.

$x=$		0		2
$f'(x)$	$+$	0	$-$	0 $+$
$f(x)$	↗		↘	↗

So $f(x)$ is decreasing on $(0,2)$

Question 4- Let $f(x) = -x^3 + 3x$. Find all critical points, and identify their type.

A) There is a local max at $x = 0$ and a local min at $x = 2$.
B) There is a local min at $x = -1$ and a local max at $x = 1$.
C) There is a local max at $x = -1$ and a local min at $x = 1$.
D) There is a local min at $x = 0$ and a local max at $x = 2$.
E) There is a local max at $x = 0$ and a saddle point at $x = 1$.

$$f'(x) = -3x^2 + 3 = -3(x^2 - 1) = -3(x+1)(x-1)$$

So there are CP's at $x=1$ and $x=-1$.

	$x=-1$		$x=1$	
$f'(x)$	$-$	0	$+$	0 $-$

So there is a min at $x=-1$ and a max at $x=1$

Question 5 Let $f(x) = x^3 + x - 1$. Determine the interval on which the graph is concave down.

A) $(-\infty, 0)$ B) $(1, 2)$ C) $(0, 2)$ D) $(1, 4)$ E) $(0, \infty)$

$f'(x) = 3x^2 + 1$

$f''(x) = 6x$

There is one possible inflection point, at $x = 0$.

$f''(x) > 0$ when $x > 0$ and

$f''(x) < 0$ when $x < 0$. So $f(x)$ is concave down on $(-\infty, 0)$

Question 6 Find all values of x for which the following function has slope equal to 0?

$$f(x) = e^{x^2 - 4x}$$

A) $x = 3$ B) $x = 2$ C) $x = 1$ D) $x = 0$ E) This function never has slope 0.

$f'(x) = e^{x^2 - 4x}(2x - 4)$

So $f'(x) = 0$ when $x = 2$

Long Answer Questions-Question 1 (12 points)

A magazine offers subscriptions at a rate of 30 dollars. They have 6,000 subscribers. It will gain 300 subscribers for every dollar it lowers the price. What price should the magazine set to maximize its revenue? You must explain why your answer is an absolute maximum.

Let x = # of 1 dollar reductions.

So # of subscribers $= 6{,}000 + 300x$

price per subscription $= 30 - x$

So revenue $= (6000 + 300x)(30 - x) = R(x)$

or $R(x) = 180{,}000 + 3000x - 300x^2$

$R'(x) = 3{,}000 - 600x$

So $x = 5$ is a CP.

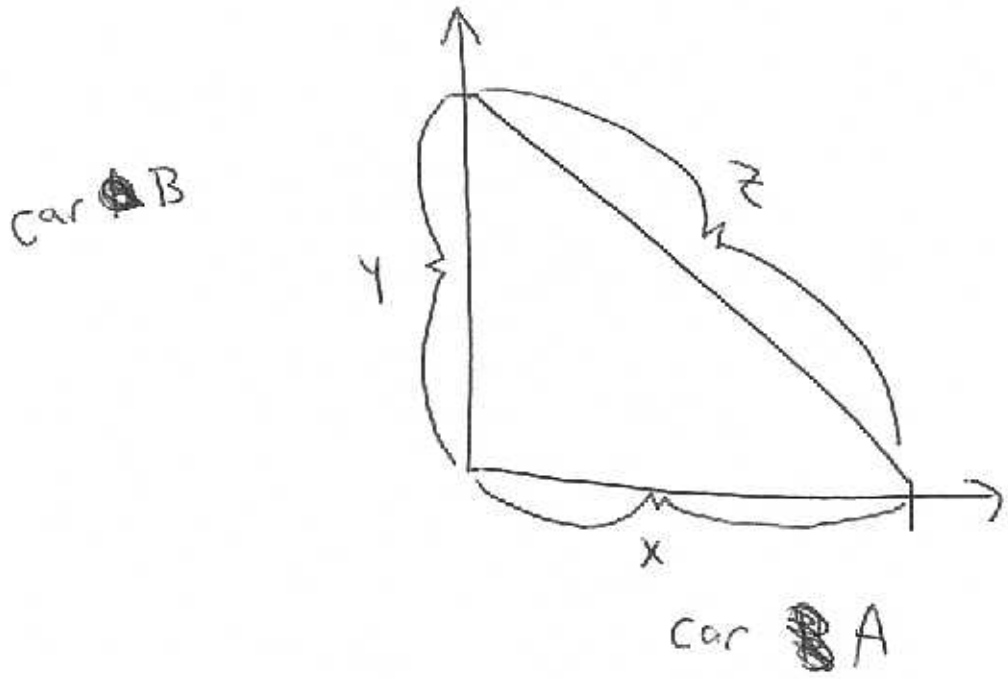
It is an absolute max, since $R(x)$ is a concave down parabola.

The optimal price is $p = 30 - 5 = 25$ dollars

Question 2 (12 points)

Two cars start from the same intersection at the same time. Car A heads east at a constant speed of 40 miles per hour. Car B heads north at a constant speed of 30 miles per hour.

- What is the distance between the two cars after two hours?
- At what rate is the distance between them changing at that time?



We have $x^2 + y^2 = z^2$. So $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

After 2 hours, $x = 80$ and $y = 60$. So by Pythagorean thm, $z = 100$.

Thus $80(40) + 60(30) = 100\frac{dz}{dt}$. So $\frac{dz}{dt} = 50$

MATH 1300-Midterm # 2-2004

NAME and I.D.# SOLUTIONS - VERSION 2

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Place your answers to the multiple choice questions in the boxes below. All your work on the long answer questions must be clearly marked. You may use the backs of pages. If **you need additional scrap paper, it will be provided by the proctors.**

NO CALCULATORS. NO BOOKS. NO NOTES.

On the long answer questions, you must show your work.

Answers:

#1	#2	#3	#4

#5	#6

Multiple Choice Section-Question 1- If $g(x) = \ln(3x^2 - 2)$, what is $g'(1)$?

A) 3 B) 2 C) 6 D) 5 E) 4

$$g'(x) = \frac{1}{3x^2-2}(6x)$$

So $g'(1) = 6$

Question 2- A certain bacteria grows exponentially. The initial population was 400,000. One hour after the experiment started, the population had grown to 900,000. What was the population a half hour after the experiment started.

A) 500,000 B) 650,000 C) 600,000 D) 700,000 E) 800,000

$P(t) = 400{,}000\,b^t \quad P(1) = 900{,}000$

So $P(1) = 900{,}000 = 400{,}000\,b^1$

So $b = \frac{9}{4}$ Thus $P(t) = 400{,}000\left(\frac{9}{4}\right)^t$

$P\left(\frac{1}{2}\right) = 400{,}000\left(\frac{3}{2}\right) = 600{,}000$

Question 3- Let $f(x) = x^3 - 3x^2 + 3$. On what interval is the function decreasing?

A) $(1, \infty)$ B) $(1, 3)$ C) $(2, 3)$ D) $(-1, 2)$ E) $(0, 2)$

$f'(x) = 3x^2 - 6x = 3x(x-2)$

The function is decreasing on $(0,2)$ as in version 1.

Question 4- Let $f(x) = x^3 - 3x$. Find all critical points, and identify their type.

A) There is a local max at $x = 0$ and a local min at $x = 2$.
B) There is a local min at $x = -1$ and a local max at $x = 1$.
C) There is a local max at $x = -1$ and a local min at $x = 1$.
D) There is a local min at $x = 0$ and a local max at $x = 2$.
E) There is a local max at $x = 0$ and a saddle point at $x = 1$.

$f'(x) = 3x^2 - 3 = 3(x^2 - 1) = 3(x+1)(x-1)$

There are 2 CP's at $x = 1$ and $x = -1$

x		-1		1	
$f'(x)$	$+$	0	$-$	0	$+$
$f(x)$	$\nearrow$		$\searrow$		$\nearrow$

There is a local max at $x = -1$ and a local min at $x = 1$

Question 5 Let $f(x) = x^3 + x - 1$. Determine the interval on which the graph is concave up.

A) $(-\infty, 0)$ **B)** $(1, 2)$ **C)** $(0, 2)$ **D)** $(1, 4)$ **E)** $(0, \infty)$

$f''(x) = 6x$

which is positive on $(0, \infty)$

Question 6 Find all values of x for which the following function has slope equal to 0?

$$f(x) = e^{x^2 - 2x}$$

A) $x = 3$ **B)** $x = 2$ **C)** $x = 1$ **D)** $x = 0$ **E)** This function never has slope 0.

$f'(x) = e^{x^2-2x}(2x-2)$ which $= 0$

when $x = 1$.

Long Answer Questions-Question 1 (12 points)

A magazine offers subscriptions at a rate of 36 dollars. They have 6,000 subscribers. It will gain 200 subscribers for every dollar it lowers the price. What price should the magazine set to maximize its revenue? You must explain why your answer is an absolute maximum.

Let x = # of one dollar reductions.

Then price/subscription $= 36 - x$

# of subscribers $= 6{,}000 + 200x$

So Revenue $= R(x) = (36 - x)(6{,}000 + 200x)$

$= 216{,}000 + 1200x - 200x^2$

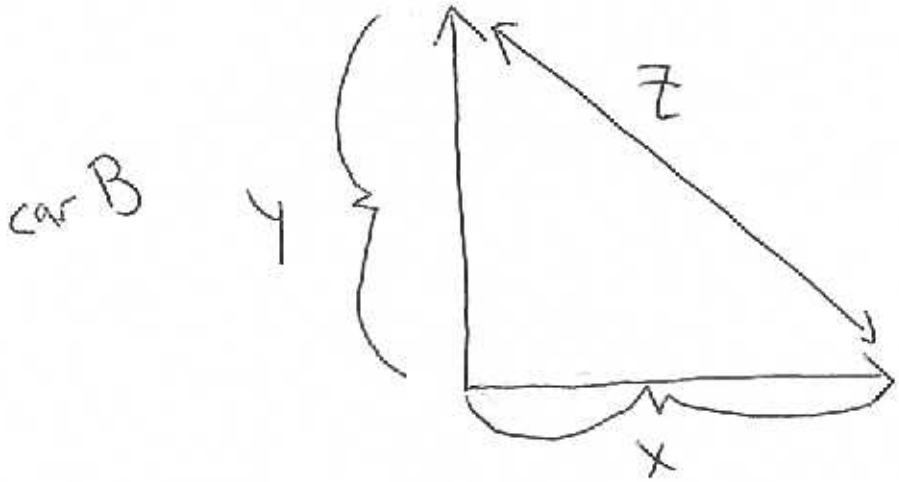
$R'(x) = 1200 - 400x$

The point $x = 3$ is an absolute max, since the function is a concave down parabola.

Question 2 (12 points)

Two cars start from the same intersection at the same time. Car A heads east at a constant speed of 40 miles per hour. Car B heads north at a constant speed of 30 miles per hour.

- What is the distance between the two cars after one hour?
- At what rate is the distance between them changing at that time?



We have $x^2 + y^2 = z^2$ or $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

Here $y = 30$
$x = 40$ So by Pythagorean Thm $z = 50$.

So $30(30) + 40(40) = 50\left(\frac{dz}{dt}\right)$

or $\frac{dz}{dt} = 50$