MATH 1300-MIDTERM # 2-2006

NAME and I.D.# Solutions - Version #2

Instructions– This exam consists of 4 multiple choice questions and 3 long answer questions. The multiple choice questions are worth 5 points each, and the long answer questions are as indicated. The total value of the exam is 60 points.

Place your answers to the multiple choice questions in the boxes below. All your work on the long answer questions must be clearly marked. You may use the backs of pages.

If you need additional scrap paper, it will be provided by the proctors.

NO CALCULATORS. NO BOOKS. NO NOTES.

ON LONG ANSWER QUESTIONS, YOU MUST SHOW YOUR WORK

Answers:

B	B	A	D
#1	#2	#3	#4

Multiple Choice Section-Question 1-
Suppose that a demand function is given by $p = \frac{100}{2x+2}$. What is the elasticity of demand when $x = 4$? Is demand elastic or inelastic?

A) $\frac{-5}{4}$, inelastic **B)** $\frac{-5}{4}$, elastic **C)** $\frac{-1}{8}$, elastic **D)** $-\frac{3}{5}$, elastic **E)** $-\frac{3}{5}$, inelastic

$x = 4, p = 10 \qquad \frac{dp}{dx} = \frac{-200}{(2x+2)^2} = \frac{-200}{100} = -2$

$\eta = \frac{10/4}{-2} = \frac{-5}{4}$

Question 2- Consider $f(x) = x^3 - 9x^2 + 24x - 3$. For what interval is this function decreasing?

A) $(-1, 3)$ **B)** $(2, 4)$ **C)** $(0, 2)$ **D)** $(3, \infty)$ **E)** $(1, 3)$

$f'(x) = 3x^2 - 18x + 24 = 3(x^2 - 6x + 8) = (x-4)(x-2)$

		2		4	
$f'(x)$	+	0	−	0	+

$f(x)$ is decreasing on $(2, 4)$

Question 3- Calculate:

$$\int (\frac{x}{\sqrt{x^2+5}})dx$$

A) $\sqrt{x^2+5}+C$ B) $2\sqrt{x^2+5}+C$ C) $\frac{1}{2}\sqrt{x^2+5}+C$ D) $\frac{1}{\sqrt{x^2+5}}+C$ E) $4\sqrt{x^2+5}+C$

$$\boxed{\begin{aligned} u &= x^2+5 \\ du &= 2x\,dx \end{aligned}}$$

$$= \frac{1}{2}\int \frac{2x\,dx}{\sqrt{x^2+5}} = \frac{1}{2}\int \frac{du}{\sqrt{u}} = \frac{1}{2}\int u^{-1/2}\,du$$

$$= \frac{1}{2}\frac{u^{1/2}}{1/2} = u^{1/2} = \sqrt{x^2+5}+C$$

Question 4- Calculate the area under the curve $f(x) = 6x^2+2x-2$ and above the interval $[1,2]$.

A) 12 B) 16 C) 8 D) 19 E) 27

$$\int_1^2 (6x^2+2x+2)\,dx$$

$$2x^3+x^2+2x\Big|_1^2 = 24-5 = 19$$

Long Answer Questions-Question 1 (14 points)

A retail store can sell 20 stereos per week at a price of 600 dollars each. The manager estimates that for each 10 dollar reduction in price, she can sell two more stereos per week. How many stereos should she sell to maximize revenue?

- Write a function for the total revenue, as a function of the number of stereos sold.
- How many stereos should she sell to maximize revenue?
- Be sure to explain why your answer is an absolute maximum.

Let x = # of stereos

x	p
20	600
22	590

$m = \frac{-10}{2} = -5$ $y = mx + b = -5x + b$

Plug in $(20, 600)$ $600 = -5(20) + b$

$b = 700$

$y = -5x + 700$

$R(x) = \text{revenue} = y \cdot x = -5x^2 + 700x$

$R'(x) = -10x + 700$. There is a CP at $x = 70$

Question 2 (12 points)
Find $\frac{dy}{dx}$ at the point (1,1) if

$$x^3 + 4xy^2 - 4 = y^4$$

Differentiating with respect to x, we get

$$3x^2 + 4y^2 + 8xy\frac{dy}{dx} = 4y^3\frac{dy}{dx}$$

Plugging in (1,1)

$$3 + 4 + \frac{8dy}{dx} = 4\frac{dy}{dx}$$

So $$4\frac{dy}{dx} = -7$$

$$\frac{dy}{dx} = \frac{-7}{4}$$

Long Answer Question 1 (10 points) Consider the function:

$$f(x) = \frac{x^3+2}{x} = x^2 + \frac{2}{x}$$

Find the domain of the function. Find all x- and y- intercepts. Find the intervals of increase and decrease. Find all critical points and determine their type. Find all inflection points and asymptotes. Determine the behavior of the function near the vertical asymptotes. On the following page, graph the function, incorporating all of this information.

- Domain $= \{x \mid x \neq 0\}$
- There is no y-intercept
- x-intercept at $\sqrt[3]{-2}$

There is a VA at $x=0$

$\lim\limits_{x \to 0^+} f(x) = +\infty$

$f'(x) = 2x - \frac{2}{x^2} \Rightarrow x = 1$ is a CP

$\lim\limits_{x \to 0^-} f(x) = -\infty$

		0		1	
$f'(x)$	$-$	UN	$-$	0	$+$

So $f(x)$ is decreasing on $(-\infty, 0)$ and $(0,1)$

increasing on $(1, \infty)$

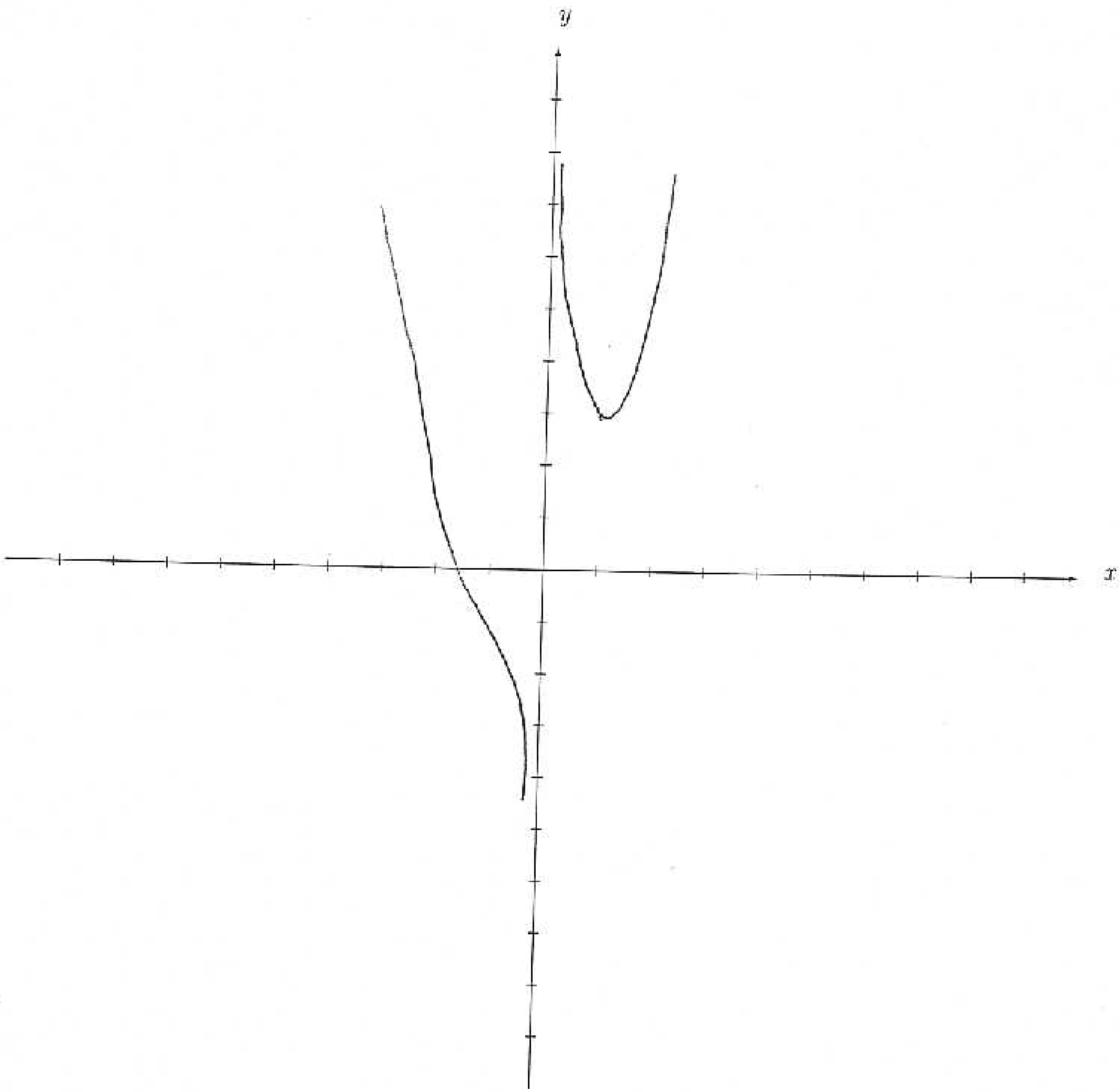
& $x=1$ is a local min.

$f''(x) = 2 + \frac{4}{x^3}$. There is a possible IP at $x = \sqrt[3]{-2}$

		$\sqrt[3]{2}$		0	
$f''(x)$	$+$	0	$-$	UN	$+$

So $x = \sqrt[3]{-2}$ is an IP

$\lim\limits_{x \to \infty} f(x) = \infty = \lim\limits_{x \to -\infty} f(x)$ So there is no HA


y
x

Space for additional work