MATH 1300-MIDTERM # 2-2004

NAME and I.D.# Solutions V1

INSTRUCTIONS– This exam consists of 6 multiple choice questions and 2 long answer questions. The multiple choice questions are worth 5 points each, and the long answer questions are as indicated. The total value of the exam is 60 points.

Place your answers to the multiple choice questions in the boxes below. All your work on the long answer questions must be clearly marked. You may use the backs of pages. If you need additional scrap paper, it will be provided by the proctors.

NO CALCULATORS. NO BOOKS. NO NOTES.

On the long answer questions, you must show your work.

Answers:

#1	#2	#3	#4
E	D	C	D

#5	#6
B	B

Multiple Choice Section-Question 1- Suppose $x^3+y^3=9xy$. Find $\frac{dy}{dx}$ at the point (2,4).

A) $\frac{3}{11}$ B) $\frac{2}{7}$ C) $\frac{11}{8}$ D) $\frac{3}{8}$ E) $\frac{4}{5}$

$$3x^2 + 3y^2\frac{dy}{dx} = 9x\frac{dy}{dx} + 9y \quad \text{(Plug in (2,4))}$$

$$12 + 48\frac{dy}{dx} = 18\frac{dy}{dx} + 36$$

$$30\frac{dy}{dx} = 24$$

$$\frac{dy}{dx} = \frac{24}{30} = \frac{4}{5}$$

Question 2- What is the absolute minimum value for the function $g(x) = x^3 - 3x^2 + 3$ on the interval $[1, 4]$

A) 4 B) 2 C) 0 D) -1 E) 6

$g'(x) = 3x^2 - 6x = 3x(x-2)$. So there are 2 CP's at $x = 0, 2$. Ignore $x=0$, since not in interval

x	g(x)
1	1
2	-1
4	19

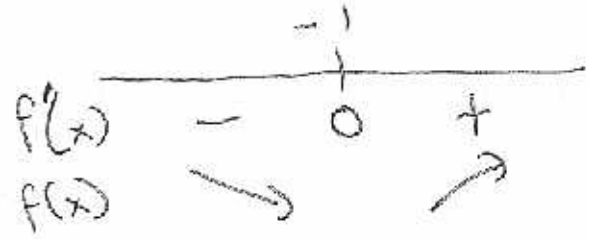
So the minimum value is -1.

Question 3- Let $f(x) = xe^x$. On what interval is the function decreasing?

A) $(-1, \infty)$ B) $(-2, 2)$ C) $(-\infty, -1)$ D) $(-3, \infty)$ E) $(-2, 3)$

$f'(x) = xe^x + e^x = (x+1)e^x$

There is one CP, at $x = -1$.



So $f(x)$ is decreasing on $(-\infty, -1)$

Question 4- Let $f(x) = 2x^3 + 3x^2 - 36x + 5$. Find all critical points, and identify their type.

A) There is a local max at $x = 0$ and a local min at $x = 2$.
B) There is a local min at $x = -1$ and a local max at $x = 1$.
C) There is a local max at $x = -1$ and a local min at $x = 1$.
D) There is a local max at $x = -3$ and a local min at $x = 2$.
E) There is a local min at $x = -3$ and a local max at $x = 2$.

$f'(x) = 6x^2 + 6x - 36 = 6(x^2 + x - 6) = 6(x+3)(x-2)$

There are 2 CP's at $x = -3$ and $x = 2$.

$f''(x) = 12x + 6$.

$f''(-3) < 0$. So there is a local max at $x = -3$

$f''(2) > 0$ So there is a local min at $x = 2$.

Question 5

Consider the function:

$$h(x) = \frac{3-x}{x-4}$$

Which of the following statements are correct?

A) $\lim_{x\to 4+} h(x) = \infty$ (B) $\lim_{x\to 4+} h(x) = -\infty$ C) $\lim_{x\to 4-} h(x) = -\infty$

D) $\lim_{x\to 3-} h(x) = -\infty$ E) $\lim_{x\to 3+} h(x) = -\infty$

$\lim_{x\to 4^+} h(x) = -\infty$ (Sample point $x = 4{,}01$)

$\lim_{x\to 4^-} h(x) = +\infty$ (Sample Point $x = 3{,}99$)

Question 6 Suppose that a demand function is given by $p = 180 - \frac{x^2}{4}$. What is the elasticity of demand when $x = 12$? Is demand elastic or inelastic?

A) $\frac{-1}{2}$, elastic (B) -2,elastic C) $\frac{-1}{3}$, elastic D) -2, inelastic E) $\frac{-1}{2}$, inelastic

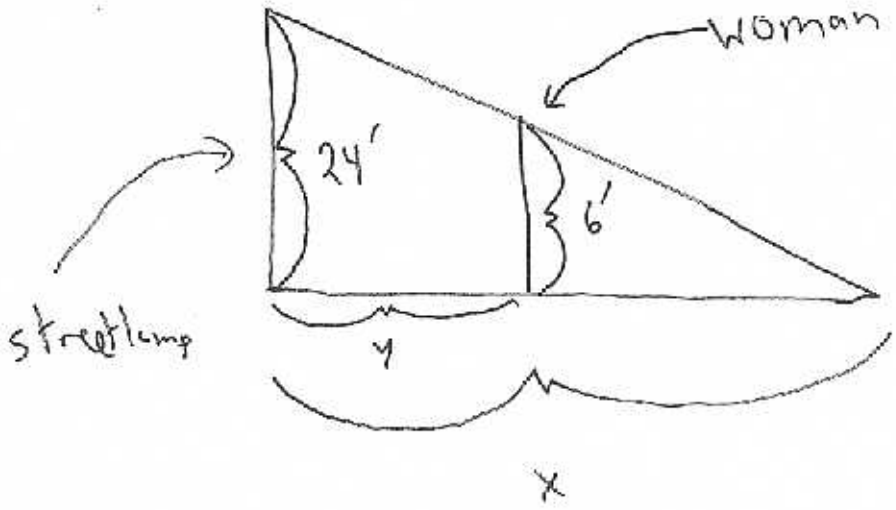
$$p = 180 - \frac{12^2}{4} = 180 - 36 = 144$$

$$p' = -\frac{x}{2}, \text{ so } p'(12) = -6.$$

$$\eta = \frac{p/x}{dp/dx} = \frac{\frac{144}{12}}{-6} = -2. \quad \text{Demand is elastic}$$

Question 2 (12 points)

A 6 foot tall woman is walking away from a 24 foot tall streetlamp at a speed of 8 feet per second. How fast is the tip of her shadow moving along the ground?



We want $\frac{dx}{dt}$. We know $\frac{dy}{dt} = 8$

By similar triangles

$$\frac{6}{24} = \frac{x-y}{x} \Rightarrow 6x = 24x - 24y$$

or $x = \frac{4}{3}y$. So $\frac{dx}{dt} = \frac{4}{3}\frac{dy}{dt} = \frac{4}{3}(8) = \frac{32}{3}$

MATH 1300-MIDTERM # 2-2004

NAME and I.D.# Solutions - V2

INSTRUCTIONS– This exam consists of 6 multiple choice questions and 2 long answer questions. The multiple choice questions are worth 5 points each, and the long answer questions are as indicated. The total value of the exam is 60 points.

Place your answers to the multiple choice questions in the boxes below. All your work on the long answer questions must be clearly marked. You may use the backs of pages. If you need additional scrap paper, it will be provided by the proctors.

NO CALCULATORS. NO BOOKS. NO NOTES.

On the long answer questions, you must show your work.

Answers:

#1	#2	#3	#4
B	E	A	C

#5	#6
A	B

Multiple Choice Section-Question 1- Suppose $x^3+y^3=9xy$. Find $\frac{dy}{dx}$ at the point (4,2).

A) $\frac{8}{11}$ B) $\frac{5}{4}$ C) $\frac{11}{8}$ D) $\frac{7}{8}$ E) $\frac{7}{5}$

$3x^2 + 3y^2 \frac{dy}{dx} = 9x\frac{dy}{dx} + 9y$ Plug in (4,2)

$48 + 12\frac{dy}{dx} = 36\frac{dy}{dx} + 18$

$30 = 24\frac{dy}{dx}$ $\frac{dy}{dx} = \frac{30}{24} = \frac{5}{4}$

Question 2- What is the absolute maximum value for the function $g(x) = x^3 - 3x^2 + 3$ on the interval $[1, 4]$

A) -1 B) 12 C) 10 D) 3 E) 19

$g'(x) = 3x^2 - 6x = 3x(x-2)$

There are 2 CP's at 0 and 2. Ignore 0.

x	g(x)
1	1
2	-1
4	19

The maximum value is 19.

Question 3- Let $f(x) = xe^x$. On what interval is the function increasing?

A) $(-1, \infty)$ B) $(-2, 2)$ C) $(-\infty, -1)$ D) $(-3, \infty)$ E) $(-2, 3)$

$f'(x) = xe^x + e^x = (x+1)e^x$. There is one CP at $x = -1$.

		-1	
$f'(x)$	$-$	0	$+$
$f(x)$	$\searrow$		$\nearrow$

$f(x)$ is increasing on $(-1, \infty)$

Question 4- Let $f(x) = 2x^3 - 9x^2 - 24x + 1$. Find all critical points, and identify their type.

A) There is a local max at $x = 0$ and a local min at $x = 2$.
B) There is a local min at $x = -1$ and a local max at $x = 4$.
C) There is a local max at $x = -1$ and a local min at $x = 4$.
D) There is a local max at $x = -3$ and a local min at $x = 2$.
E) There is a local min at $x = -3$ and a local max at $x = 2$.

$f'(x) = 6x^2 - 18x - 24 = 6(x^2 - 3x - 4) = 6(x-4)(x+1)$

There are 2 CP's at $x = -1, 4$

$f''(x) = 12x - 18$

$f''(-1) < 0$. So there is a local max at $x = -1$

$f''(4) > 0$ So there is a local min at $x = 4$

Question 5

Consider the function:

$$h(x) = \frac{x-3}{x-4}$$

Which of the following statements are correct?

A) $\lim_{x\to 4^+} h(x) = \infty$ B) $\lim_{x\to 4^+} h(x) = -\infty$ C) $\lim_{x\to 4^-} h(x) = \infty$

D) $\lim_{x\to 3^-} h(x) = -\infty$ E) $\lim_{x\to 3^-} h(x) = -\infty$

$\lim_{x\to 4^+} h(x) = +\infty$ sample point $x = 4.01$

$\lim_{x\to 4^-} h(x) = -\infty$ sample point $x = 3.99$

Question 6 Suppose that a demand function is given by $p = 50 - \frac{x^2}{8}$. What is the elasticity of demand when $x = 4$? Is demand elastic or inelastic?

A) $\frac{-1}{5}$, elastic B) -12, elastic C) $\frac{-1}{3}$, elastic D) -12, inelastic E) $\frac{-1}{5}$, inelastic

When $x = 4$, $p = 48$.

$p'(x) = -\frac{x}{4}$. $p'(4) = -1$

So $\eta = \frac{\frac{48}{4}}{-1} = -12$. Demand is elastic

Long Answer Questions-Question 1 (18 points)

A manufacturer can produce tapes at a cost of 3 dollars each. It currently sells them at a price of 5 dollars each. At this price, consumers buy 6000 tapes per month. For each 1 dollar increase in price, 600 fewer tapes will be sold.

- What is the demand function?
- What is the profit function?
- How many units should the producer sell to maximize profit? Be sure to explain why your answer is an absolute maximum.

x = # of units, p = price/unit

Revenue $= xp$

p is linear

x	p
6000	5
5400	6

$m = \frac{\Delta p}{\Delta x} = \frac{-1}{600}$

$y = mx+b = -\frac{1}{600}x+b$ Plug in $(6,000, 5)$

$5 = -10+b \Rightarrow b = 15 \Rightarrow y = -\frac{1}{600}x+15$

Revenue $= xp = -\frac{1}{600}x^2+15x$

Cost $= 3x$

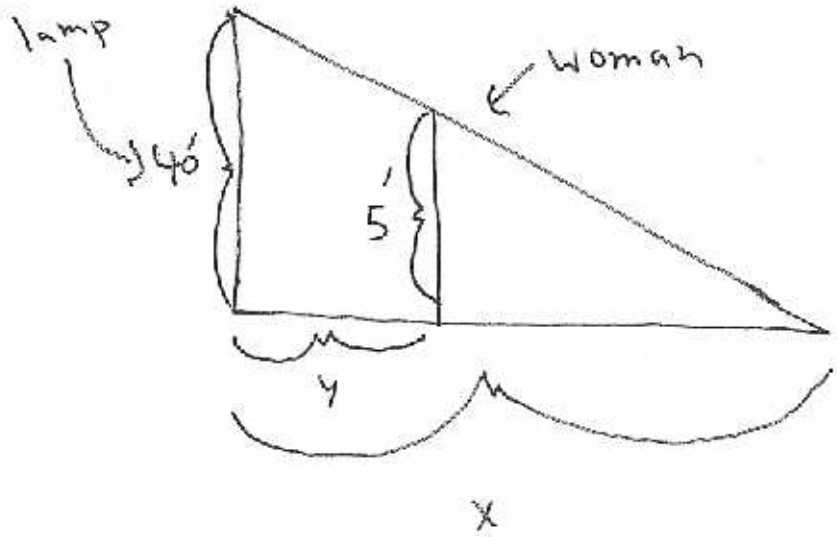
Profit $= P(x) = -\frac{1}{600}x^2+15x-3x = -\frac{1}{600}x^2+12x$

$P'(x) = -\frac{1}{300}x+12 \Rightarrow x = 3600$ is a CP

It is an absolute max, since $P(x)$ is a concave down parabola.

Question 2 (12 points)

A 5 foot tall woman is walking away from a 40 foot tall streetlamp at a speed of 6 feet per second. How fast is the tip of her shadow moving along the ground?



We want $\frac{dx}{dt}$. We know $\frac{dy}{dt} = 6$

By similar triangles,

$$\frac{5}{40} = \frac{x-y}{x} \Rightarrow 5x = 40x - 40y$$

or $-35x = -40y$ or $x = \frac{8}{7}y$. So $\frac{dx}{dt} = \frac{8}{7}\frac{dy}{dt} = \frac{8}{7} \cdot 6$

$$= \frac{48}{7}$$