

MATH 1300B-MIDTERM1-2003

Multiple Choice Section-Question 1-

Which of the following functions is the inverse of $f(x) = \frac{2x+5}{3x-4}$?

- A) $f^{-1}(x) = \frac{4x+5}{3x-2}$ B) $f^{-1}(x) = \frac{3x-4}{2x+5}$ C) $f^{-1}(x) = \frac{5-2x}{4+3x}$ D) $f^{-1}(x) = \frac{5x-2}{4x-3}$
E) $f^{-1}(x) = \frac{3-4x}{5+2x}$

Solution: (A)

In $f(x) = \frac{2x+5}{3x-4}$ we replace x by y and $f(x)$ by x , then

$$x = \frac{2y + 5}{3y - 4}.$$

Solve y we get

$$y = f^{-1}(x) = \frac{4x + 5}{3x - 2}.$$

Question 2- Consider the function $f(x) = \frac{x^3 - x^2 - 9x - 7}{x + 1}$. Which of the following statements is true?

- A) $\lim_{x \rightarrow -1} f(x)$ does not exist B) $\lim_{x \rightarrow -1} f(x) = 4$ C) $\lim_{x \rightarrow -1} f(x) = -4$
D) $\lim_{x \rightarrow -1} f(x) = 1$ E) $\lim_{x \rightarrow -1} f(x) = 0$

Solution: (C)

The numerator can be written as

$$x^3 - x^2 - 9x - 7 = (x + 1)(x^2 - 2x - 7).$$

Thus

$$\lim_{x \rightarrow -1} f(x) = \lim_{x \rightarrow -1} (x^2 - 2x - 7) = -4.$$

Question 3- Describe the interval(s) on which the following function

$$f(x) = \frac{x + 2}{x(x + 1)}$$

is continuous:

- A) $(0, \infty)$ B) $(-1, 0)$ C) $(-\infty, -1) \cup (-1, 0) \cup (0, \infty)$
D) $(-\infty, -1) \cup (-1, 0) \cup (0, \infty)$ E) none of the above

Solution: (C)

Since denominator can not be 0, so

$$x \neq 0, -1.$$

Question 4- Solve the following equation for x :

$$e^{4x^2-1} = 1.$$

- A) $x = 1/4$ B) $x = \pm 1/4$ C) $x = \pm 1/2$ D) $x = \pm 2$ E) none of the above

Solution: (C)

Since $e^0 = 1$, so $4x^2 - 1 = 0$. Solving this we obtain $x = \pm 1/2$.

Question 5- Which one of the following is NOT equal to x ?

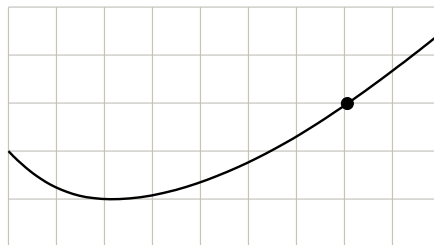
- A) $e^{\ln(x-x^2)-\ln(1-x)}$ B) $\ln(e^{3x}) + \ln\left(\frac{1}{e^x}\right)^2$ C) $\frac{1}{\pi}e^{\ln(\pi+x)}$ D) $2 \ln[(\sqrt{e})^x]$ E) $2e^{\ln x - \ln 2}$

Solution: (C)

Since

$$\frac{1}{\pi}e^{\ln(\pi+x)} = \frac{1}{\pi}(\pi+x) = 1 + \frac{x}{\pi}.$$

Question 6



Which one of the following best approximates the slope of the tangent line to the graph at the dot?

- A) 2 B) 1/2 C) -2 D) -1/2 E) 1

Solution: (B)

Since the slope is

$$\frac{2.5}{3.5}.$$

Long Answer Questions—Question 1 (13 points)

(Your answer should be a mathematical expression that can be readily plugged into a calculator to obtain an answer.)

- a) (6 points) You are planning to buy a house. You need a balance of \$35,000 four years from now. If the account you wish to open pays the annual interest rate of 1.8% that is compounded monthly, how much should your opening balance be? (Assume that you will not be making additional deposits after opening.)

- b) (7 points) If, instead, the interest was compounded continuously, then how much should your opening balance be?

Solution: We have

$$A = 35000, r = 1.8\% = 0.018, n = 12, t = 4.$$

- a) From

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

we obtain

$$P = 35000(1.0015)^{-48}.$$

- b) From

$$A = e^{rt}$$

we obtain

$$P = 35000(1.0015)^{-0.072}.$$

Question 2 (12 points)

Let $f(x) = 3x^2 - 7$.

a) (6 points) Use the definition of the derivative to compute $f'(2)$.

b) (6 points) Compute the equation of the tangent line at $(2, f(2))$.

Solution: a) From the definition of the derivative, we have

$$\begin{aligned} f'(2) &= \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \rightarrow 0} \frac{[3(2+h)^2 - 7] - [3(2)^2 - 7]}{h} \\ &= \lim_{h \rightarrow 0} (12 + 3h) = 12. \end{aligned}$$

b) Note that $f(2) = 3(2)^2 - 7 = 5$, we get

$$y - 5 = 12(x - 2).$$

Simplify this yields

$$y = 12x - 19.$$

Space for additional work