MATH 1300-MIDTERM # 1-2006

NAME and I.D.# Solutions - Version 1

Instructions - This exam consists of 4 multiple choice questions and 3 long answer questions. The multiple choice questions are worth 6 points each, and the long answer questions are as indicated. The total value of the exam is 60 points.

Place your answers to the multiple choice questions in the boxes below. All your work on the long answer questions must be clearly marked. You may use the backs of pages.

FOR LONG ANSWER QUESTIONS, YOU MUST SHOW YOUR WORK

NO CALCULATORS. NO BOOKS. NO NOTES.

If you need additional scrap paper, it will be provided by the proctors.

Answers:

#1	#2	#3	#4
E	A	C	B

Multiple Choice Section-Question 1-
Calculate:

$$\lim_{x \to 3} \frac{x^4 - 81}{x - 3}$$

A) 48 **B)** 4 **C)** 8 **D)** 16 **E)** 108

$x^4 - 81 = (x^2+9)(x^2-9) = (x^2+9)(x+3)(x-3)$

So $\lim\limits_{x \to 3} \frac{x^4-81}{x-3} = \frac{(x^2+9)(x+3)(x-3)}{(x-3)} = \lim\limits_{x \to 3} (x^2+9)(x+3) = 108$

Question 2- Find the equation of the tangent line of the function $f(x) = \sqrt{x^2 + 4x - 1}$ at $x = 1$.

A) $y = \frac{3x}{2} + \frac{1}{2}$ **B)** $y = \frac{x}{7} + \frac{13}{7}$ **C)** $y = \frac{2x}{5} + \frac{8}{5}$

D) $y = 3x - 1$ **E)** $y = 4x - 2$

$f'(x) = \frac{1}{2}(x^2+4x-1)^{-1/2}(2x+4)$

$f'(1) = \frac{1}{2}(4)^{-1/2}(6) = \frac{1}{2} \cdot \frac{1}{2} \cdot 6 = \frac{3}{2}$

Note $f(1) = 2$.

So $y = mx+b \Rightarrow y = \frac{3}{2}x + b$ and $2 = \frac{3}{2} + b$ or $b = \frac{1}{2}$

Question 3

Let $f(x) = (x^2+1)^3(2x-3)$. Find $f'(x)$ at $x = 1$.

A) -64 B) 36 C) -8 D) -44 E) -68

$f'(x) = (x^2+1)^3(2) + 3(x^2+1)^2(2x)(2x-3)$

$f'(1) = 8(2) + 3(4)(2)(-1) = -8$

Question 4 If the production costs of x units of a product is $C(x) = 14x + 110$ and the revenues from the sale of x units of that same product is $R(x) = 2x^2 - 50$, how many units must be produced before the company beaks even? Note that x must be greater than 0.

A) 2 B) 10 C) 4 D) 12 E) 18

$C(x) = R(x) \Rightarrow 14x + 110 = 2x^2 + 50 \Rightarrow 2x^2 - 14x - 60 = 0$

or $x^2 - 7x - 30 = 0$

or $(x-10)(x+3) = 30$

so $x = 10$

Long Answer Questions-Question 1 (12 points)

Using only the definition of derivative as a limit, calculate $f'(x)$ where

$$f(x) = 3x^2 + x - 2$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{3(x+\Delta x)^2 + (x+\Delta x) - 2 - (3x^2 + x - 2)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{3(x^2 + 2x\Delta x + \Delta x^2) + (x+\Delta x) - 2 - 3x^2 - x + 2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{3x^2 + 6x\Delta x + 3\Delta x^2 + x + \Delta x - 2 - 3x^2 - x + 2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{6x\Delta x + 3\Delta x^2 + \Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} 6x + 3\Delta x + 1$$

$$= 6x + 1$$

Question 2 (12 points)

Suppose that a deposit of 4,000 dollars is made into a bank that gives 6% interest. Suppose that interest is compounded continuously.

- (2 points) Write a formula for $A(t)$, the value of the investment, after t years.
- (2 points) How much will the investment be worth after 7 years?
- (8 points) How long will it take for the investment to double?

A) $A(t) = Pe^{rt} = 4000\, e^{.06t}$

B) $A(7) = 4000\, e^{.06(7)} = 4000\, e^{.42}$

C) We must solve

$$8000 = 4000\, e^{.06t}$$

So $2 = e^{.06t}$

$$\ln(2) = \ln\left(e^{.06t}\right) = .06t$$

So $t = \dfrac{\ln(2)}{.06}$

Question 3 (12 points)

The demand function for a certain good is

$$p = D(x) = -\frac{1}{160}x + 10,$$

where x is the number of items consumers are willing to purchase when the price is p dollars. Furthermore, the production cost for x items is given by

$$C(x) = 8\sqrt{x} + 1000.$$

- (3 points) Write the formula for $R(x)$, the revenue from the sale of x items.
- (3 points) Write the formula for $P(x)$, the profit from the sale of x items.
- (3 points) Find the gain in profit obtained by increasing production from 16 to 17 items.
- (3 points) Find the marginal profit at $x = 16$ items.

A) $R(x) = xp = xD(x) = -\frac{1}{160}x^2 + 10x$

B) $P(x) = R(x) - C(x) = -\frac{1}{160}x^2 + 10x - 8\sqrt{x} - 1000$

C) $\text{Gain} = P(17) - P(16) = -\frac{1}{160}(17)^2 + 10(17) - 8(\sqrt{17}) - 1000$

$-\left[-\frac{1}{160} \cdot 16^2 + 10(16) - 8(4) - 1000\right]$

$= -\frac{1}{160}(17^2 - 16^2) + 10(17-16) - 8(\sqrt{17} - 4)$

You can simplify a bit more, if you want

D) Marginal Profit $= P'(x)$. $P'(x) = -\frac{x}{80} + 10 - \frac{4}{\sqrt{x}}$

$P'(16) = -\frac{1}{5} + 10 - 1 = \frac{44}{5}$

MATH 1300-MIDTERM # 1-2006

NAME and I.D.# Solutions - Version 2

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Place your answers to the multiple choice questions in the boxes below. All your work on the long answer questions must be clearly marked. You may use the backs of pages.

FOR LONG ANSWER QUESTIONS, YOU MUST SHOW YOUR WORK

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Answers:

E	A	C	D
#1	#2	#3	#4

Multiple Choice Section-Question 1-
Calculate:

$$\lim_{x\to 2} \frac{x^4 - 16}{x - 2}$$

A) 2 B) 4 C) 8 D) 16 E) 32

$x^4 - 16 = (x^2+4)(x^2-4) = (x^2+4)(x+2)(x-2)$

$\lim_{x\to 2} \frac{x^4-16}{x-2} = \lim_{x\to 2} \frac{(x^2+4)(x+2)(x-2)}{x-2} = \lim_{x\to 2} (x^2+4)(x+2) = 32$

Question 2- Find the equation of the tangent line of the function $f(x) = \sqrt{x^2 + x + 2}$ at $x = 1$.

A) $y = \frac{3x}{4} + \frac{5}{4}$ B) $y = \frac{x}{7} + \frac{13}{7}$ C) $y = \frac{2x}{5} + \frac{8}{5}$

D) $y = 3x - 1$ E) $y = 4x - 2$

$f'(x) = \frac{1}{2}(x^2+x+2)^{-1/2}(2x+1)$

$f'(1) = \frac{1}{2} \cdot \frac{1}{2} \cdot 3 = \frac{3}{4}$

$f(1) = 2$ So $y = \frac{3x}{4} + \frac{5}{4}$

Question 3

Let $f(x) = (x^2 + 1)^3(3x - 2)$. Find $f'(x)$ at $x = 1$.

A) 64 B) 36 C) 48 D) 72 E) 128

$f'(x) = 3(x^2+1)^2(2x)(3x-2) + (x^2+1)^3(3)$

$f'(1) = 3(4)(2)(1) + 8(3) = 24 + 24 = 48$

Question 4 If the production costs of x units of a product is $C(x) = 20x + 98$ and the revenues from the sale of x units of that same product is $R(x) = 2x^2 + 50$, how many units must be produced before the company beaks even? Note that x must be greater than 0.

A) 2 B) 6 C) 4 D) 12 E) 18

$C(x) = R(x) \Rightarrow 20x + 98 = 2x^2 + 50 \Rightarrow$

$2x^2 - 20x - 48 = 0 \Rightarrow$

$x^2 - 10x - 24 = 0 \Rightarrow (x-12)(x+2) = 0$

$\Rightarrow x = 12$

Long Answer Questions-Question 1 (12 points)

Using only the definition of derivative as a limit, calculate $f'(x)$ where

$$f(x) = 2x^2 + 2x + 3$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{[2(x+\Delta x)^2 + 2(x+\Delta x) + 3] - [2x^2 + 2x + 3]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{[2x^2 + 4x\Delta x + \Delta x^2 + 2x + 2\Delta x + 3] - 2x^2 - 2x - 3}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{4x\Delta x + 4\Delta x^2 + 2\Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} 4x + 4\Delta x + 2$$

$$= 4x + 2$$

Question 2 (12 points)

Suppose that a deposit of 5,000 dollars is made into a bank that gives 4% interest. Suppose that interest is compounded continuously.

- (2 points) Write a formula for $A(t)$, the value of the investment, after t years.
- (2 points) How much will the investment be worth after 5 years?
- (8 points) How long will it take for the investment to triple?

A) $A(t) = 5{,}000\, e^{.04t}$ (using $A(t) = P_0 e^{rt}$)

B) $A(5) = 5{,}000\, e^{.04(5)} = 5{,}000\, e^{.2}$

C) We must solve

$$\cancel{1}0{,}000 = 5{,}000\, e^{.04t}$$

So $$2 = e^{.04t}$$

$$\ln(2) = .04t$$

$$t = \frac{\ln(2)}{.04}$$

Question 3 (12 points)

The demand function for a certain good is

$$p = D(x) = -\frac{1}{500}x + 16,$$

where x is the number of items consumers are willing to purchase when the price is p dollars. Furthermore, the production cost for x items is given by

$$C(x) = 5\sqrt{x} + 1000.$$

- (3 points) Write the formula for $R(x)$, the revenue from the sale of x items.
- (3 points) Write the formula for $P(x)$, the profit from the sale of x items.
- (3 points) Find the gain in profit obtained by increasing production from 25 to 26 items.
- (3 points) Find the marginal profit at $x = 25$ items.

A) $R(x) = x D(x) = \frac{-1}{500}x^2 + 16x$

B) $P(x) = R(x) - C(x) = \frac{-1}{500}x^2 + 16x - (5\sqrt{x} + 1000)$

C) Gain $= P(26) - P(25)$

$= \left[\frac{-1}{500}26^2 + 16\cdot 26 - 5\sqrt{26} - 1000\right] - \left[\frac{-1}{500}25^2 + 16\cdot 25 - 5\cdot 5 - 1000\right]$

$= \frac{-1}{500}(26^2 - 25^2) + 16\cdot 1 - 5(\sqrt{26} - 5)$

D) Marginal Profit $= P'(x) = \frac{-x}{250} + 16 - \frac{5}{2\sqrt{x}}$

$P'(25) = \frac{-1}{10} + 16 - \frac{1}{2}$