MATH 1300-MIDTERM # 2-2005

NAME and I.D.# Solutions, Version 1

INSTRUCTIONS– This exam consists of 6 multiple choice questions and 2 long answer questions. The multiple choice questions are worth 5 points each, and the long answer questions are as indicated. The total value of the exam is 60 points.

Place your answers to the multiple choice questions in the boxes below. All your work on the long answer questions must be clearly marked. You may use the backs of pages. **If you need additional scrap paper, it will be provided by the proctors.**

NO CALCULATORS. NO BOOKS. NO NOTES.

On the long answer questions, you must show your work.

Answers:

#1	#2	#3	#4	#5	#6
E	D	E	B	B	E

Multiple Choice Section-Question 1-
Suppose that a demand function is given by $p = -x^2 - 2x + 21$. What is the elasticity of demand when $x = 3$? Is demand elastic or inelastic?

A) $\frac{-5}{7}$, inelastic **B)** $\frac{-5}{7}$, elastic **C)** $\frac{-1}{4}$, elastic **D)** $\frac{-3}{5}$, elastic **E)** $\frac{-1}{4}$, inelastic

$\eta = \frac{p/x}{dp/dx}$ Here $p = -9-6+21 = 6$

$dp/dx = -2x-2 = -6-2 = -8$

So $\eta = \frac{6/3}{-8} = -\frac{1}{4}$. Demand is inelatstic

Question 2- Consider $f(x) = x^3 + 9x^2 - 24x + 3$. For what interval is this function concave up?

A) $(-\infty, -3)$ **B)** $(3, \infty)$ **C)** $(-\infty, 3)$ **D)** $(-3, \infty)$ **E)** $(-3, 3)$

$f'(x) = 3x^2 + 18x - 24$

$f''(x) = 6x + 18$ $f''(-3) = 0$

$f''(x)$ − −3 +

$f(x)$ CD CU

Answer: $(-3, \infty)$

Question 3- What is the absolute minimum value for the function $g(x) = \frac{1}{5-x}$ on the interval $[0, 2]$

A) 1 **B)** 0 **C)** $\frac{1}{3}$ **D)** 4 **E)** $\frac{1}{5}$

$g'(x) = \frac{1}{(5-x)^2}$.

There are no CP's.

$g(0) = 1/5 \leftarrow$ absolute min

$g(2) = 1/3$

Question 4- Let $f(x) = \frac{1}{(1-x)^2}$. On what interval is the function increasing?

A) $(-2, 3)$ **B)** $(-\infty, 1)$ **C)** The function is never increasing. **D)** $(-\infty, \infty)$ **E)** $(1, \infty)$

$f(x) = (1-x)^{-2} \Rightarrow f'(x) = \frac{2}{(1-x)^3}$

$f'(x)$

$x=1$

$+$ UN $-$

So $f(x)$ is increasing on $(-\infty, 1)$

Question 5

Let $f(x) = xe^{3x}$. Find all critical points, and identify their type.

A) There is a local max at $x = -1/3$.
B) There is a local min at $x = -1/3$.
C) There is a local min at $x = 1$.
D) There is a local min at $x = 3$.
E) There is a local max at $x = 3$.

$$f'(x) = e^{3x} + 3xe^{3x} = (1+3x)e^{3x}$$

There is a CP at $x = -1/3$

		$x = -1/3$	
$f'(x)$	$-$	0	$+$
$f(x)$	$\searrow$		$\nearrow$

So $x = -1/3$ is a local min.

Question 6 Consider the function:

$$h(x) = \frac{x-1}{x-5}$$

Which of the following statements are correct?

A) $\lim_{x\to1^+} h(x) = -\infty$ B) $\lim_{x\to1^+} h(x) = \infty$ C) $\lim_{x\to1^-} h(x) = -\infty$
D) $\lim_{x\to5^+} h(x) = -\infty$ E) $\lim_{x\to5^-} h(x) = -\infty$

$$\lim_{x\to5^+} h(x) = \frac{+}{+} = +\infty$$

Test point 5,01

$$\lim_{x\to5^-} h(x) = \frac{+}{-} = -\infty$$

Test point 4,99

Long Answer Questions-Question 1 (15 points)

An apple orchard currently has 13 trees, and produces 51 pounds of apples per tree. For each tree added to the orchard, the amount produced per tree decreases by 3 pounds.

- Write a function for the number of pounds produced for the entire orchard.
- How many trees should there be to maximize the total number of pounds of apples the orchard produces?
- Be sure to explain why your answer is an absolute maximum.

Let x = # of additional trees.

Then - Total # of trees $= 13 + x$

Pounds / tree $= 51 - 3x$

So Total pounds $= (13 + x)(51 - 3x)$

$T(x) = (13+x)(51-3x) = 663 + 12x - 3x^2$

$T'(x) = 12 - 6x$, So $x = 2$ is a CP.

It is an absolute max, since $T(x)$ is a concave down parabola.

Total # of trees $= 13 + 2 = 15$

Question 2 (15 points)

The initial number of wild rabbits on an island was estimated to be 140,000. After 2 years, the population was 210,000. Assume the population grows exponentially.

- Find the function $P(t)$ which gives the population after t years.
- How many years will it take for the population to be 490,000?

$$P(t) = P_0 b^t = 140{,}000\, b^t$$

$$P(2) = 140{,}000\, b^2 = 210{,}000$$

$$b^2 = \frac{3}{2}$$

$$b = \sqrt{3/2}$$

A) So $P(t) = 140{,}000 \left(\sqrt{3/2}\right)^t = 140{,}000 \left(3/2\right)^{t/2}$

B) Solve $490{,}000 = 140{,}000 \left(3/2\right)^{t/2}$

$$\frac{7}{2} = \left(\frac{3}{2}\right)^{t/2}$$

$$\ln\left(\frac{7}{2}\right) = \ln\left[\left(3/2\right)^{t/2}\right] = t/2\, \ln(3/2)$$

$$\Rightarrow \frac{t}{2} = \frac{\ln(7/2)}{\ln(3/2)} \Rightarrow t = \frac{2\ln(7/2)}{\ln(3/2)}$$