

FINAL EXAM-MATH 1300
FALL TERM, 2010

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Name(Print LEGIBLY) _____

I.D. Number _____

Instructions-This final examination consists of 10 multiple choice questions worth 3 points each. Your answers to the multiple choice questions must be clearly marked in the squares below. There are also 5 long answer questions worth a total of 70 points. For the long answer questions, you must show your work **on the exam itself** and clearly display your answers. **Do not unstaple these pages.**

NO CALCULATORS. NO BOOKS. NO NOTES.

**TURN OFF YOUR CELL PHONES AND
PUT THEM AWAY.**

Multiple Choice Answers:

#1

#2

#3

#4

#5

#6

#7

#8

#9

#10

Question 1- Calculate:

$$\lim_{x \rightarrow 2} \frac{x^4 - 16}{x - 2}$$

A) 9 B) 8 C) 17 D) 32 E) 72

Question 2- The function $y = f(x)$ is defined implicitly by

$$4xy^2 + 3x^2y = 7$$

Find $\frac{dy}{dx}$ at the point $(1, 1)$.

A) $-\frac{10}{11}$ B) $-\frac{13}{7}$ C) $\frac{1}{13}$ D) $-\frac{2}{17}$ E) $\frac{10}{3}$

Question 3- Find the equation of the tangent line to the graph of $y = \sqrt{8x + 1}$ when $x = 3$.

- A) $y = \frac{4}{5}x + \frac{3}{5}$ B) $y = \frac{4}{5}x + \frac{6}{5}$ C) $y = \frac{4}{5}x + \frac{13}{5}$ D) $y = \frac{4}{5}x - \frac{3}{5}$ E) $y = \frac{4}{5}x + \frac{8}{5}$

Question 4- What are the critical points of the function $f(x) = xe^{2x}$?

- A) f has no critical points B) $x = 0$ C) $x = 0, x = -\frac{1}{2}$ D) $x = -\frac{1}{2}$ E) $x = \frac{1}{4}, x = -\frac{1}{2}$

Question 5- Calculate:

$$\int_0^{\ln(3)} x e^{2x} dx$$

- A)** 0 **B)** 1 **C)** $\frac{9}{2} \ln(3) - 2$ **D)** $\frac{1}{2} \ln(3) - 3$ **E)** $\frac{5}{2} \ln(3) + 1$

Question 6- Evaluate:

$$\int_0^1 \frac{3}{x^2} dx$$

- A)** 1 **B)** -1 **C)** 3 **D)** -3 **E)** divergent

Question 7- Suppose $f'(x) = \frac{4}{\sqrt{x}}$ and $f(1) = 3$. Find $f(9)$.

- A) 13 B) 12 C) 19 D) 37 E) 51

Question 8- If the demand function is $D(x) = -x^2 - x + 25$ and the supply function is $S(x) = x^2 + 2x - 2$, find the producer surplus.

- A) 41 B) 11 C) $\frac{9}{2}$ D) 27 E) $\frac{4}{3}$

Question 9- If $f(x, y) = y^2e^{xy}$, what is $f_{xy}(\ln(2), 1)$?

- A) $3 \ln(2)$ B) $6 + 2 \ln(2)$ C) $5 \ln(2)$ D) $5 + \ln(2)$ E) $4 + 3 \ln(2)$

Question 10- If $f(x, y) = e^{3x} + 2x - 2y^3$, how many critical points does $f(x, y)$ have?

- A) 0 B) 1 C) 2 D) 3 E) 4

Long Answer Question 1 (14 points)

Recall that radioactive substances decay exponentially, and that the *half-life* of a radioactive substance is the amount of time it takes for half of the substance to decay. Suppose a radioactive substance has a half-life of 6 years.

- If I begin with 11 grams of the substance, how much will I have after 5 years?
- How long does it take for 9 grams of the substance to decay to 1 gram?

Long Answer Question 2 (14 points)

Calculate the following two indefinite integrals:

$$\int \frac{xe^{x^2}}{\sqrt{e^{x^2} + 2}} dx$$

$$\int x^2 \ln(x) dx$$

Long Answer Question 3 (14 points)

The manager of an apple orchard wants to harvest her apples at a time that will maximize revenue. Currently apples are selling for 90 cents per pound. The price will decrease by 2 cents per pound for each week she waits. Each tree in the orchard now has 33 pounds of apples, but for each week she waits, each tree will gain one additional pound of apples.

- Write a formula for the total revenue for the orchard?
- How many weeks should she wait to produce the maximum revenue?

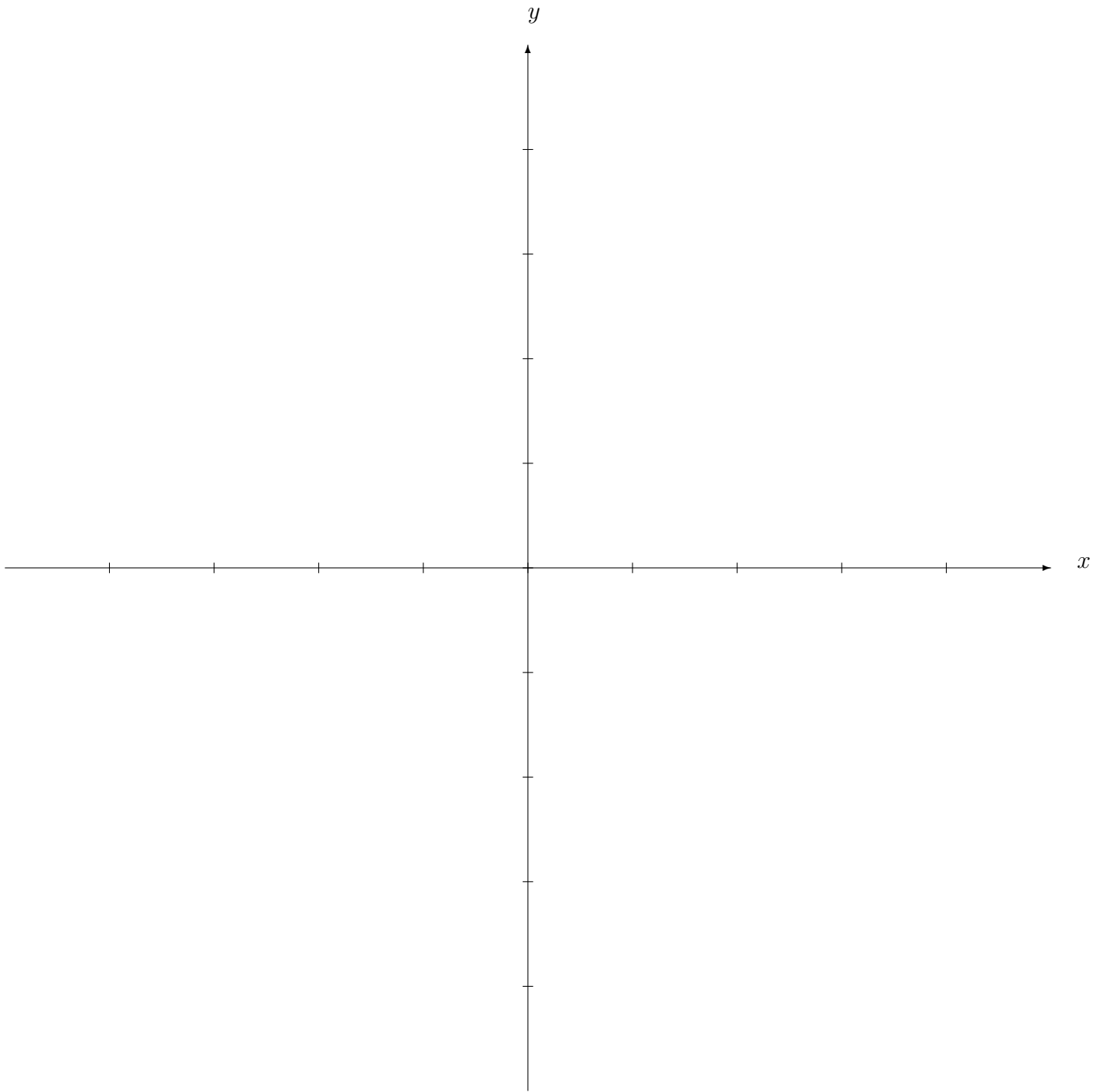
Be sure to explain why your answer is an absolute maximum.

Long Answer Question 4 (16 points)

Consider the two functions:

$$f(x) = 4 - x^2 \text{ and } g(x) = 3x$$

- (a) **(4 points)** Find the intersection points of the graphs of the two functions.
- (b) **(6 points)** On the next page, graph these functions, and shade the region between the graphs of f and g for $x \in [0, 4]$.
- (c) **(6 points)** Find the area of the shaded region.



Long Answer Question 5 (12 points)

Consider the function of two variables $f(x, y) = \frac{x^3}{3} + y^2 - 3x - 2xy$.

- (a) **(3 points)** Calculate the first-order partial derivatives.
- (b) **(3 points)** Find all critical points.
- (c) **(6 points)** Identify what type of critical points they are (local max, local min or saddle point).

Extra page for additional work