MATH 1300-MIDTERM # 1-2009

NAME and I.D.#____

Instructions: This midterm exam consists of 4 multiple choice questions and 3 long answer questions. The multiple choice questions are worth 5 points each, and the long answer questions are as indicated. The total value of the exam is 60 points.

Place your answers to the multiple choice questions in the boxes below. All your work on the long answer questions must be clearly marked. You may use the backs of pages.

For long answer questions, YOU MUST SHOW YOUR WORK

NO CALCULATORS. NO BOOKS. NO NOTES.

If you need additional scrap paper, it will be provided by the proctors.

Answers:

$\square$	$\square$	$\square$	$\square$
#1	#2	#3	#4

Multiple Choice Section Questions (1-4)

Question 1 *Calculate:*

$$\lim_{x \to 3} \frac{x^3 - 27}{x - 3}$$

A) 9 B) 18 C) 27 D) 36, E) The limit does not exist.

$x^3 - 27 = (x-3)(x^2 + 3x + 9)$ So

$\lim_{x \to 3} \frac{x^3 - 27}{x - 3} = \lim_{x \to 3} x^2 + 3x + 9 = 9 + 9 + 9 = 27$

Question 2 *Find the equation of the tangent line of the function $f(x) = (2x+1)\sqrt{3x+1}$ at $x = 1$.*

A) $y = \frac{25x}{4} - \frac{1}{4}$ B) $y = \frac{x}{9} + \frac{2}{9}$ C) $y = \frac{25x}{4} - \frac{7}{4}$ D) $y = \frac{x}{9} + \frac{2}{9}$ E) $y = \frac{25x}{2} - \frac{1}{2}$

$f'(x) = 2\sqrt{3x+1} + (2x+1)\frac{3}{2\sqrt{3x+1}}$

$f'(1) = 2 \cdot \sqrt{4} + 3 \cdot \frac{3}{2 \cdot \sqrt{4}} = 4 + \frac{9}{4} = \frac{25}{4}$

$f(1) = 3\sqrt{4} = 6$

$y = \frac{25}{4}x + b \Rightarrow 6 = \frac{25}{4} \cdot 1 + b \Rightarrow b = -\frac{1}{4}$

$\Rightarrow y = \frac{25x}{4} - \frac{1}{4}$.

Question 3 *Use implicit differentiation to find* $\frac{dy}{dx}$ *at the point (2,1) for the equation:*

$$x^2y + 2xy^2 = 8$$

A) $\frac{1}{3}$ B) $\frac{2}{5}$ C) $\frac{4}{3}$ D) $\frac{-1}{2}$ E) $\frac{-2}{3}$

$[x^2y + 2xy^2]' = [8]' \Rightarrow (2xy + x^2y') + (2y^2 + 4xyy') = 0$

$\Rightarrow 2 \cdot 2 \cdot 1 + 2^2 y' + 2 \cdot 1^2 + 4 \cdot 2 \cdot 1 \cdot y' = 0$

$\Rightarrow 4 + 4y' + 2 + 8y' = 0$

$\Rightarrow 12y' = -6 \Rightarrow y' = -\frac{6}{12} = -\frac{1}{2}$

Question 4 *Find the inverse of the function:*

$$f(x) = \frac{2x-1}{x+3}$$

A) $\frac{-3x-1}{x-2}$ B) $\frac{2x-3}{x-2}$ C) $\frac{3x-1}{x-3}$ D) $\frac{-3x-1}{2x+3}$ E) $\frac{2x-1}{3x+1}$

$x = \frac{2y-1}{y+3} \Rightarrow x(y+3) = 2y-1 \Rightarrow xy + 3x = 2y - 1$

$\Rightarrow xy - 2y = -1 - 3x$

$\Rightarrow y(x-2) = -1 - 3x$

$\Rightarrow y = \frac{-1-3x}{x-2}$

Long Answer Section Questions (5-7)

Question 5 (12 points) *Using only the definition of derivative as a limit, calculate $f'(x)$ where*

$$f(x) = (3 - 2x)^2$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{(3-2(x+\Delta x))^2 - (3-2x)^2}{\Delta x}$$

Now: $(3 - 2(x+\Delta x))^2 = 9 - 12(x+\Delta x) + 4(x+\Delta x)^2$
$= 9 - 12x - 12\Delta x + 4x^2 + 8x\Delta x + 4(\Delta x)^2$

and $(3-2x)^2 = 9 - 12x + 4x^2$

So $(3-2(x+\Delta x))^2 - (3-2x)^2 =$

$(9 - 12x - 12\Delta x + 4x^2 + 8x\Delta x + 4(\Delta x)^2) - (9 - 12x + 4x^2)$

$= -12\Delta x + 8x\Delta x + 4(\Delta x)^2$

So the limit becomes

$$f'(x) = \lim_{\Delta x \to 0} \frac{-12\Delta x + 8x\Delta x + 4(\Delta x)^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} -12 + 8x + 4\Delta x = \underline{-12 + 8x}.$$

(2 pts for definition, 3 pts for plugging in, 4 pts for calculation, 3 pts for limit)

Question 6 (14 points) *Suppose that a deposit of 3,000 \$ is made into a bank that gives 5% interest. Suppose that interest is compounded 6 times per year.*

(a) *(2 points) Write a formula for $A(t)$, the value of the investment, after t years in this case.*

(b) *(4 points) How much will the investment be worth after 3 years?*

(c) *(8 points) How long will it take for the investment to triple?*

(a) $A(t) = P \cdot \left(1 + \frac{r}{n}\right)^{n \cdot t}$ which becomes

$= P \cdot \left(1 + \frac{0.05}{6}\right)^{6t}$

$= 3{,}000 \left(1 + \frac{0.05}{6}\right)^{6t}$

(1 pt for formula, 1 pt for substitution)

(b) Plug in $t = 3$:

$A(3) = 3{,}000 \left(1 + \frac{0.05}{6}\right)^{18}$

(2 pt for choosing correct formula, 2 pt for substituting)

(c) $A(t) = 3 \cdot P = 9000 \Rightarrow$

$9000 = 3000 \left(1 + \frac{0.05}{6}\right)^{6t}$ — 3 pts

$\Rightarrow 3 = \left(1 + \frac{0.05}{6}\right)^{6t}$

$\Rightarrow \ln 3 = 6t \cdot \ln\left(1 + \frac{0.05}{6}\right)$ — 3 pts

$\Rightarrow t = \dfrac{\ln 3}{6 \ln\left(1 + \frac{0.05}{6}\right)}$ years — 2 pts

Question 7 (14 points)

- *(8 points) A business sells 5,000 radios per month at a price of 300 dollars each. It is estimated that monthly sales will increase by a level of 50 units for each 2 dollar decrease in price. Find the demand function, as well as the revenue function.*
- *(6 points) Suppose another business makes burglar alarms. Suppose that there is a initial fixed cost of 25,000 dollars. Suppose each alarm costs 90 dollars to make and that the manufacturer has set a price of 150 dollars per alarm. Write down a profit and cost function, and determine the breakeven point.*

Demand function $p = D(x)$ linear

goes through $(300, 5000)$ and $(300-2, 5000+50) = (298, 5050)$

Slope $m = \frac{\Delta y}{\Delta x} = \frac{2}{-50} = -\frac{1}{25}$

So $p = -\frac{x}{25} + b$ plug in $(300, 5000)$

$300 = \frac{-5000}{25} + b = -200 + b \Rightarrow b = 500$

$\Rightarrow p = -\frac{x}{25} + 500$

$\Rightarrow R(x) = xD(x) = x \cdot p = -\frac{x^2}{25} + 500x$.

$C(x) = 25{,}000 + 90x$

$R(x) = 150x$

$\Rightarrow P(x) = R(x) - C(x) = 60x - 25{,}000$

Break-Even:

$P(x) = 0 \Rightarrow$

$60x = 25{,}000 \Rightarrow$

$x = \frac{2500}{6}$ units.

Bonus Question (1 point): Name the two historical figures who can both claim to have invented calculus.

Newton & Leibniz.

Space for additional work