

FINAL EXAM-MATH 1300
FALL TERM, 2006

R. Blute & P. Boily

Name(Print LEGIBLY)_____

I.D. Number_____

Instructions-This final examination consists of 10 multiple choice questions worth 3 points each. Your answers to the multiple choice questions must be clearly marked in the squares below. There are also 5 long answer questions worth a total of 70 points. For the long answer questions, you must show your work **on the exam itself** and clearly display your answers. **Do not unstaple these pages.**

NO CALCULATORS. NO NOTES OR BOOKS.

Multiple Choice Answers:

#1

#2

#3

#4

#5

#6

#7

#8

#9

#10

Question 1- Find the equation of the tangent line to the graph of $y = \frac{2}{x^2-2}$ when $x = 2$.

- A) $y = -2x + 6$ B) $y = -2x + 5$ C) $y = -2x + 1$ D) $y = 2x - 1$ E) $y = 2x - 3$

Question 2- Suppose $x^3 + 4xy^2 - 2 = 3y^4$ Find $\frac{dy}{dx}$ at the point $(1, 1)$.

- A) $-\frac{1}{5}$ B) 1 C) $\frac{7}{4}$ D) $\frac{4}{9}$ E) -1

Question 3- On what interval is the function $g(x) = -2x^3 + 15x^2 - 36x + 3$ increasing?

- A) $(2, \infty)$ B) $(2, 3)$ C) $(-1, 1)$ D) $(-2, 2)$ E) $(-2, 4)$

Question 4- Which of the following statements is correct for the function $g(x) = xe^x$.

- A) $x = -1$ is a local min. B) $x = -1$ is a local max. C) $x = -3$ is a local min.
D) $x = 1$ is a local min. E) $x = 1$ is a local max.

Question 5- Calculate:

$$\int_1^2 (x^3 - 3x^2 + 4x - 1)dx$$

- A) $\frac{4}{5}$ B) $\frac{1}{2}$ C) $\frac{4}{3}$ D) $\frac{7}{4}$ E) $\frac{8}{3}$

Question 6- Suppose that for a certain product, the demand function is given by $D(x) = (x - 5)^2$ and the supply function is given by $S(x) = x^2 + x + 3$. Calculate the consumer surplus.

- A) 11 B) $\frac{4}{7}$ C) $\frac{9}{2}$ D) $\frac{7}{5}$ E) $\frac{44}{3}$

Question 7- If $f(x)$ is a function such that $f'(x) = 4x^3 - 6x - 8$ and $f(2) = 2$, find $f(1)$.

- A) 0 B) 1 C) 2 D) 3 E) 4

Question 8- Calculate:

$$\int_1^{\infty} \frac{x dx}{x^2 + 1}$$

- A) $\frac{3}{4}$ B) $\frac{1}{64}$ C) $\frac{1}{2}$ D) $\frac{5}{4}$ E) The integral diverges.

Question 9- If $f(x, y) = x^2y + 3y^2x^3 - 4x$, calculate $f_y(2, 1)$
A) -9 B) 52 C) -11 D) 76 E) 27

Question 10- If $f(x, y) = 2yx^2 - xy^3 - y^2 + 7$, calculate $f_{xy}(1, 1)$.
A) 9 B) -2 C) 1 D) -13 E) -7

Long Answer Question 1 (12 points)

Recall that radioactive substances decay exponentially, and that the *half-life* of a radioactive substance is the amount of time it takes for half of the substance to decay. Suppose a radioactive substance has a half-life of 5 years.

- If I begin with 7 grams of the substance, how much will I have after 3 years?
- How long does it take for 6 grams of the substance to decay to 1 gram?

Question 2 (16 points)

Calculate the following two indefinite integrals:

$$\int x^4 \sqrt{x^5 + 8} dx$$

$$\int x^3 \ln(x) dx$$

Question 3 (10 points)

For the following function, find the *absolute* maximum and minimum on the interval $[0,3]$:

$$f(x) = \frac{1}{3}x^3 + 2x^2 - 5x + 1$$

Question 4 (16 points)

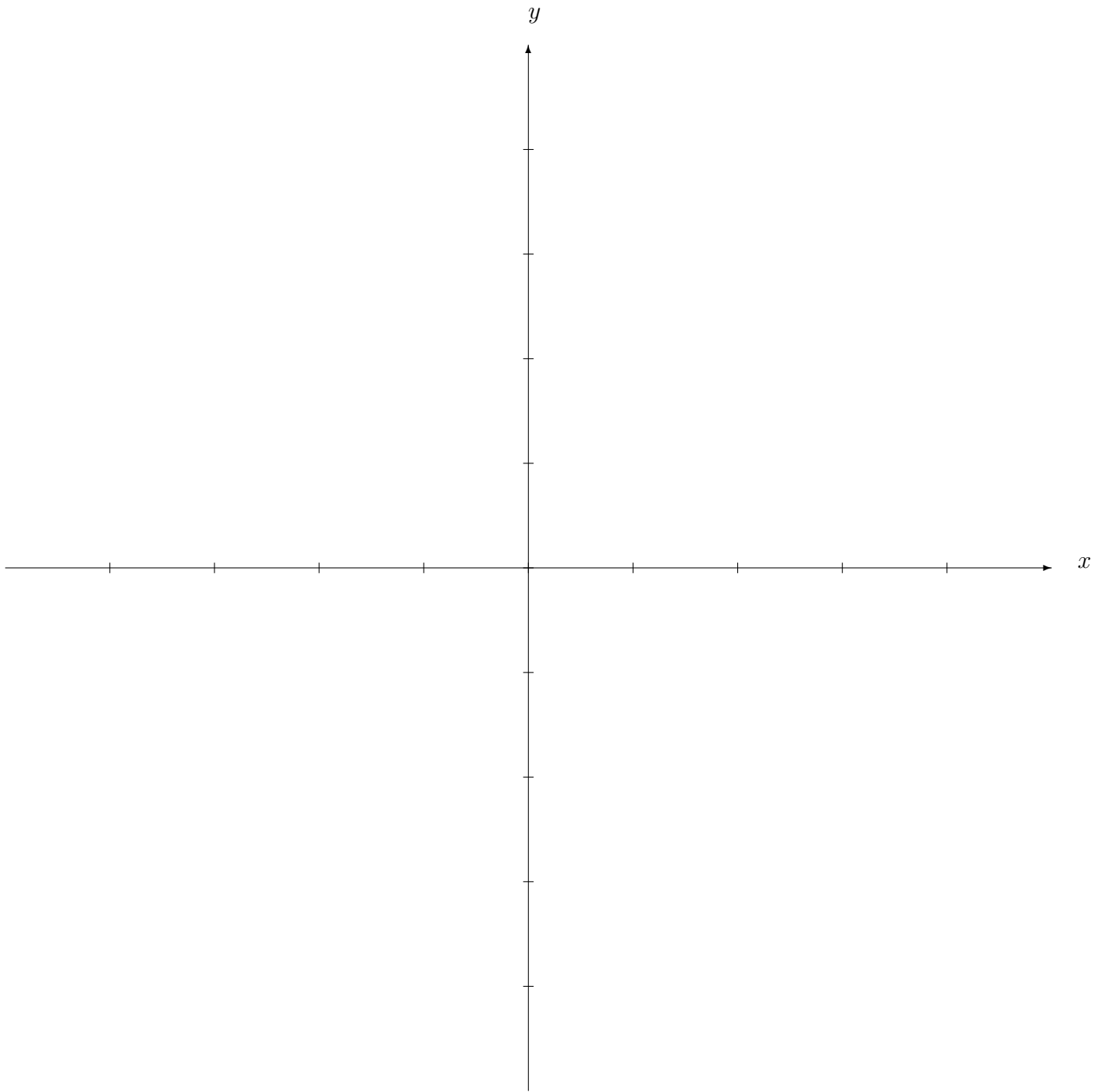
Consider the two functions:

$$f(x) = 4 - x^2 \text{ and } g(x) = x^2 - 4$$

(a)(4 points) Find the intersection points of the graphs of the two functions.

(b)(6 points) On the next page, graph these functions, and shade the region bounded by $f(x)$, $g(x)$, $x = 0$ and $x = 4$.

(c)(6 points) Find the area of the shaded region.



Question 5 (16 points)

Consider the function of two variables $f(x, y) = \frac{x^3}{3} + y^2 - 3x - 2xy$.

- Calculate the first-order partial derivatives.
- Find all critical points.
- Identify what type of critical points they are (local max, local min or saddle point).

Extra page for additional work