MATH 1300-MIDTERM # 2(a)-2007

(/60)

NAME and I.D.# (Write Clearly)________________________________

Instructions: This midterm exam consists of 4 multiple choice questions and 3 long answer questions. The multiple choice questions are worth 5 points each, and the long answer questions are as indicated. The total value of the exam is 60 points.

Place your answers to the multiple choice questions in the boxes below. All your work on the long answer questions must be clearly marked. You may use the backs of pages.

For long answer questions, YOU MUST SHOW YOUR WORK

NO CALCULATORS. NO BOOKS. NO NOTES.

If you need additional scrap paper, it will be provided by the proctors.

Answers:

#1	#2	#3	#4
C	C	A	D

Multiple Choice Section Questions (1-4)

Question 1 *Consider* $f(x) = 2x^3 + 3x^2 - 12x + 1$. *For what interval is this function decreasing?*

A) $(-\infty, -2)$ **B)** $(-2, 0)$ $\boxed{\textbf{C)}\ (-2, 1)}$ **D)** $(1, 3)$ **E)** $(1, \infty)$

Solution

$f'(x) = 6x^2 + 6x - 12 = 6(x^2 + x - 2) = 6 \cdot (x-1)(x+2)$, $x = -2$, $x = 1$

x	$-\infty$		-2		1		$+\infty$
$f'(x)$		$+$	0	$-$	0	$+$	
$f(x)$		$\nearrow$		$\searrow$		$\nearrow$	

Question 2 *Which of the following statements is correct for the function*

$$f(x) = 2x^3 + 3x^2 - 12x + 1$$

A) There is a local max at $x = -\frac{1}{2}$.
$\boxed{\textbf{C)}}$ There is an inflection point at $x = -\frac{1}{2}$.
B) There is a local min at $x = -\frac{1}{2}$.
D) There is an inflection point at $x = -2$.
E) There is an inflection point at $x = 1$.

Solution

$f'(x) = 6(x^2 + x - 2)$

$f''(x) = 6(2x + 1)$, $f''(x) = 0$, $x = -\frac{1}{2}$

x		$-1/2$	
$f''(x)$	$-$	0	$+$
	$\cap$	IP	$\cup$

Question 3 *Let $f(x)$ be a function satisfying $f'(x) = 3x^2 + 4x - 2$ and $f(1) = 2$. Find $f(2)$.*

$\boxed{\text{A) 13}}$ B) 9 C) -7 D) 23 E) -4

Solution

$$f(x) = \int f'(x)\,dx = \int (3x^2 + 4x - 2)\,dx = x^3 + 2x^2 - 2x + C$$

$$f(1) = 1 + 2 - 2 + C = 1 + C, \quad f(1) = 2 \Rightarrow 1 + C = 2, \quad C = 1$$

$$f(x) = x^3 + 2x^2 - 2x + 1$$

$$f(2) = 8 + 8 - 4 + 1 = 13$$

Question 4 *Suppose that a demand function is given by $p = \frac{50}{2x-1}$. What is the elasticity of demand when $x = 5$? Is demand elastic or inelastic?*

A) $-\frac{4}{5}$, elastic B) $-\frac{4}{5}$, inelastic C) $-\frac{2}{50}$, elastic

$\boxed{\text{D) } -\frac{9}{10}\text{, inelastic}}$ E) $-\frac{9}{10}$, elastic

Solution

$$\eta = \frac{p(x)}{x \cdot p'(x)} = \frac{50}{(2x-1) \cdot x \cdot \frac{-50 \cdot 2}{(2x-1)^2}} = -\frac{2x-1}{2 \cdot x}$$

$$x = 5, \quad \eta = -\frac{9}{10}, \quad |\eta| = \frac{9}{10} < 1$$

Long Answer Section Questions (5-7)

Question 5 (14 points)
A retail store can sell 40 toasters per week at a price of 300 \$ each. The manager estimates that for each 10 \$ reduction in price, she can sell two more toasters per week. How many toasters should she sell to maximize revenue?

a) *Write a function for the total revenue, as a function of the number of toasters sold.*

b) *How many toasters should she sell to maximize revenue?*

c) *What price per toaster maximizes the revenue?*

d) *Be sure to explain why your answer is an absolute maximum.*

Solution

a) $R(x) = x \cdot D(x)$,

x	40	42	44 ...
D	300	290	280 ...

$\Rightarrow \quad \dfrac{D-300}{x-40} = \dfrac{290-300}{42-40} = -5 \Rightarrow$

$D = 300 - 5x + 200 = 500 - 5x$

$R(x) = 500x - 5x^2$

b) $R'(x) = 500 - 10x = 0, \quad x = 50; \quad R''(x) = -10$ so $R''(50) < 0$

Therefore, at $x = 50$, $R(x)$ has local max.

As $x = 50$ is the only C.P. of $R(x)$ in $[0, \infty)$ and it is local max, it is global max.

c) $D(50) = 500 - 5 \cdot 50 = 250\$$

d) $x=50$ is the only C.P of $R(x)$ in $[0,\infty)$; as $x=50$ is local max it follows it is absolute max.

Question 6 **(12 points)** *Evaluate the following indefinite integrals:*

$$I = \int \frac{x^2}{\sqrt{x^3 - 15}} dx$$

$$I = \int \frac{1}{\sqrt{u}} \cdot \frac{1}{3} du = \frac{1}{3} \int u^{-\frac{1}{2}} du = \frac{1}{3} \frac{1}{-\frac{1}{2}+1} u^{-\frac{1}{2}+1} + C$$

$$\begin{cases} u = x^3 - 15 \\ du = u' \cdot dx = 3x^2 dx - \\ x^2 dx = \frac{1}{3} du \end{cases}$$

$$= \frac{2}{3} (x^3 - 15)^{1/2} + C$$

$$I = \int \frac{x-1}{\sqrt{x}} dx$$

$$I = \int \left(x \cdot x^{-\frac{1}{2}} - x^{-\frac{1}{2}}\right) dx = \int x^{1/2} dx - \int x^{-\frac{1}{2}} dx$$

$$= \frac{1}{1+\frac{1}{2}} x^{\frac{1}{2}+1} - \frac{1}{-\frac{1}{2}+1} x^{-\frac{1}{2}+1} + C$$

$$= \frac{2}{3} x^{\frac{3}{2}} - 2 x^{\frac{1}{2}} + C$$

Question 7 **(14 points)** *Consider the function:*

$$f(x) = \frac{x+2}{x-3}$$

Find the domain of the function. Find all x- and y- intercepts. Find the intervals of increase and decrease. Find all critical points and determine their type. Find all inflection points and asymptotes. Determine the behavior of the function near the vertical asymptotes. On the following page, graph the function, incorporating all of this information.

- $D(f) = \{x \neq 3\}$

- $f(0) = -\frac{2}{3}$, $(0, -\frac{2}{3})$; $f(x) = 0$, $x+2=0$, $x=-2$, $(-2, 0)$

- $f'(x) = \dfrac{(x-3)-(x+2)}{(x-3)^2} = -\dfrac{5}{(x-3)^2}$

x	$-\infty$	3	$+\infty$
$f'(x)$	$-$	$\Vert$	$-$
$f(x)$	$\searrow$	$\Vert$	$\searrow$

No C.P. as there is no x s.th. $f'(x) = 0$.

- $f''(x) = \dfrac{10}{(x-3)^3}$;

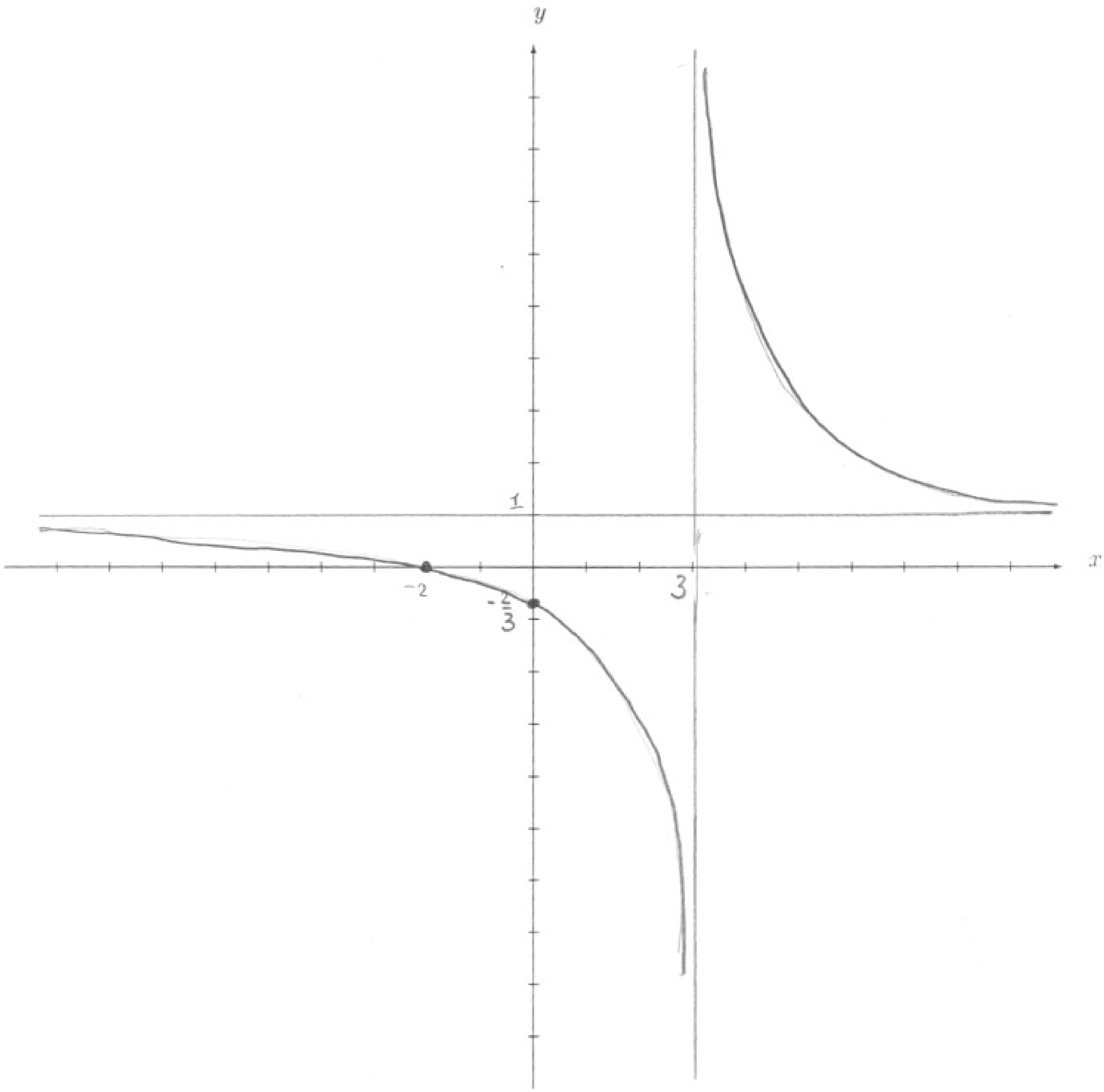
sign of $f''(x)$

x	$-\infty$	3	$+\infty$
$f''(x)$	$-$	$\Vert$	$+$
$f(x)$	$\frown$	$\Vert$	$\smile$

at $x=3$ $f''(x)$ changes sign but $x=3$ is not I.P. because $f'(3)$ is not defined

- V.A. $x = 3$, $\lim\limits_{x \to 3^{\pm}} f(x) = \dfrac{5}{\pm 0} = \pm\infty$

H.A.: $y = 1$


y
x
1
-2
$-\frac{2}{3}$
3