Exercises

7.1. Consider preferences defined over the nonnegative orthant by $(x_1, x_2) \succ (y_1, y_2)$ iff $x_1+x_2 < y_1+y_2$. Do these preferences exhibit local nonsatiation? If these are the only two consumption goods and the consumer faces positive prices, will the consumer spend all of his income? Explain.

7.2. A consumer has a utility function $u(x_1, x_2) = \max\{x_1, x_2\}$. What is the consumer's demand function for good 1? What is his indirect utility function? What is his expenditure function?

7.3. A consumer has an indirect utility function of the form

$$v(p_1, p_2, m) = \frac{m}{\min\{p_1, p_2\}}.$$

What is the form of the expenditure function for this consumer? What is the form of a (quasiconcave) utility function for this consumer? What is the form of the demand function for good 1?

7.4. Consider the indirect utility function given by

$$v(p_1, p_2, m) = \frac{m}{p_1 + p_2}.$$

(a) What are the demand functions?

(b) What is the expenditure function?

(c) What is the direct utility function?

7.5. A consumer has a direct utility function of the form

$$U(x_1, x_2) = u(x_1) + x_2.$$

Good 1 is a discrete good; the only possible levels of consumption of good 1 are $x_1 = 0$ and $x_1 = 1$. For convenience, assume that $u(0) = 0$ and $p_2 = 1$.

(a) What kind of preferences does this consumer have?

(b) The consumer will definitely choose $x_1 = 1$ if p_1 is strictly less than what?

(c) What is the algebraic form of the indirect utility function associated with this direct utility function?

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7.6. A consumer has an indirect utility function of the form $v(\mathbf{p}, m) = A(\mathbf{p})m$.

(a) What kind of preferences does this consumer have?

(b) What is the form of this consumer's expenditure function, $e(\mathbf{p}, u)$?

(c) What is the form of this consumer's indirect money metric utility function, $\mu(\mathbf{p}; \mathbf{q}, m)$?

(d) Suppose instead that the consumer had an indirect utility function of the form $v(\mathbf{p}, m) = A(\mathbf{p})m^b$ for $b > 1$. What will be the form of the consumer's indirect money metric utility function now?

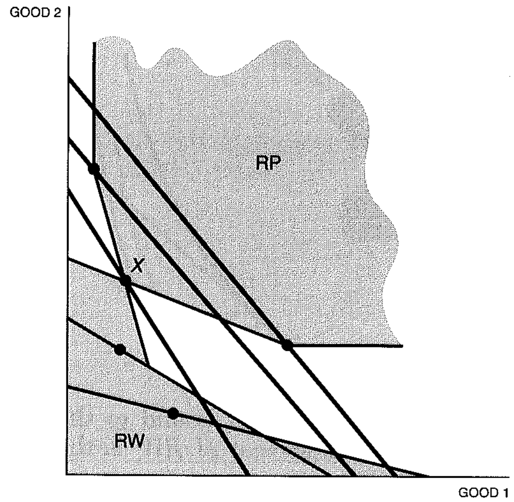

Figure 8.9 **Inner and outer bounds.** When there are several observations, the inner and outer bounds can be quite tight.

in Hurwicz & Uzawa (1971). The idea of revealed preferences is due to Samuelson (1948). The approach taken here follows that of Afriat (1967) and Varian (1982a). The derivation of the Slutsky equation using revealed preference follows Yokoyama (1968).

Exercises

8.1. Frank Fisher's expenditure function is $e(\mathbf{p}, u)$. His demand function for jokes is $x_j(\mathbf{p}, m)$, where $\mathbf{p}$ is vector of prices and $m \gg 0$ is his income. Show that jokes are a normal good for Frank if and only if $\partial^2 e/\partial p_j \partial u > 0$.

8.2. Calculate the substitution matrix for the Cobb-Douglas demand system with two goods. Verify that the diagonal terms are negative and the cross-price effects are symmetric.

8.3. Suppose that a consumer has a linear demand function $x = ap + bm + c$. Write down the differential equation you would need to solve to find the money metric utility function. If you can, solve this differential equation.

8.4. Suppose that a consumer has a semi-log demand function $\ln x = ap + bm + c$. Write down the differential equation you would need to solve to find the money metric utility function. If you can, solve this differential equation.

8.5. Find the demanded bundle for a consumer whose utility function is $u(x_1, x_2) = x_1^{\frac{3}{2}} x_2$ and her budget constraint is $3x_1 + 4x_2 = 100$.

8.6. Use the utility function $u(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{3}}$ and the budget constraint $m = p_1 x_1 + p_2 x_2$ to calculate $\mathbf{x}(\mathbf{p}, m)$, $v(\mathbf{p}, m)$, $\mathbf{h}(\mathbf{p}, u)$ and $e(\mathbf{p}, u)$.

8.7. Extend the previous exercise to the case where $u(x_1, x_2) = (x_1 - \alpha_1)^{\beta_1}(x_2 - \alpha_2)^{\beta_2}$ and check the symmetry of the matrix of substitution terms $\left(\dfrac{\partial h_j(\mathbf{p}, u)}{\partial p_i}\right)$.

8.8. Repeat the previous exercise using $u^*(x_1, x_2) = \frac{1}{2} \ln x_1 + \frac{1}{3} \ln x_2$ and show that all the previous formulae hold provided u is replaced by e^{u^*}.

8.9. Preferences are represented by $u = \phi(\mathbf{x})$ and a expenditure function, indirect utility function and demands are calculated. If the same preferences are now represented by $u^* = \psi(\phi(\mathbf{x}))$ for a monotone increasing function $\psi(\cdot)$, show that $e(\mathbf{p}, u)$ is replaced by $e(\mathbf{p}, \psi^{-1}(u^*))$, $v(\mathbf{p}, m)$ by $\psi(v(\mathbf{p}, m))$, and $\mathbf{h}(\mathbf{p}, u)$ by $\mathbf{h}(\mathbf{p}, \psi^{-1}(u^*))$. Also, check that the Marshallian demands $\mathbf{x}(\mathbf{p}, m)$ are unaffected.

8.10. Consider a two-period model with Dave's utility given by $u(x_1, x_2)$ where x_1 represents his consumption during the first period and x_2 is his second period's consumption. Dave is endowed with $(\overline{x}_1, \overline{x}_2)$ which he could consume in each period, but he could also trade present consumption for future consumption and vice versa. Thus, his budget constraint is

$$p_1 x_1 + p_2 x_2 = p_1 \overline{x}_1 + p_2 \overline{x}_2,$$

where p_1 and p_2 are the first and second period prices respectively.

(a) Derive the Slutsky equation in this model. (Note that now Dave's income depends on the value of his endowment which, in turn, depends on prices: $m = p_1 \overline{x}_1 + p_2 \overline{x}_2$.)

(b) Assume that Dave's optimal choice is such that $x_1 < \overline{x}_1$. If p_1 goes down, will Dave be better off or worse off? What if p_2 goes down?

(c) What is the rate of return on the consumption good?

8.11. Consider a consumer who is demanding goods 1 and 2. When the price of the goods are $(2, 4)$, he demands $(1, 2)$. When the prices are $(6, 3)$, he demands $(2, 1)$. Nothing else of significance changed. Is this consumer maximizing utility?

8.12. Suppose that the indirect utility function takes the form $v(p, y) = f(p)y$. What is the form of the expenditure function? What is the form of the indirect compensation function, $\mu(p; q, y)$ in terms of the function $f(\cdot)$ and y?

8.13. The utility function is $u(x_1, x_2) = \min\{x_2 + 2x_1, x_1 + 2x_2\}$.

(a) Draw the indifference curve for $u(x_1, x_2) = 20$. Shade the area where $u(x_1, x_2) \geq 20$.

(b) For what values of p_1/p_2 will the unique optimum be $x_1 = 0$?

(c) For what values of p_1/p_2 will the unique optimum $x_2 = 0$?

(d) If neither x_1 nor x_2 is equal to zero, and the optimum is unique, what must be the value of x_1/x_2?

8.14. Under current tax law some individuals can save up to $2,000 a year in an Individual Retirement Account (I.R.A.), a savings vehicle that has an especially favorable tax treatment. Consider an individual at a specific point in time who has income Y, which he or she wants to spend on consumption, C, I.R.A. savings, S_1, or ordinary savings S_2. Suppose that the "reduced form" utility function is taken to be:

$$U(C, S_1, S_2) = S_1^\alpha S_2^\beta C^\gamma.$$

(This is a reduced form since the parameters are not truly exogenous taste parameters, but also include the tax treatment of the assets, etc.) The budget constraint of the consumer is given by:

$$C + S_1 + S_2 = Y,$$

and the limit that he or she can contribute to the I.R.A. is denoted by L.

(a) Derive the demand functions for S_1 and S_2 for a consumer for whom the limit L is *not* binding.

(b) Derive the demand function for S_1 and S_2 for a consumer for whom the limit L *is* binding.

8.15. If leisure is an inferior good, what is the slope of the supply function of labor?

8.16. A utility-maximizing consumer has strictly convex, strictly monotonic preferences and consumes two goods, x_1 and x_2, each of which has a price of 1. He cannot consume negative amounts of either good. The consumer has an income of m every year. His current level of consumption is (x_1^*, x_2^*), where $x_1^* > 0$ and $x_2^* > 0$. Suppose that next year he will be given a grant of $g_1 \leq x_1^*$ which must be spent entirely on good 1. (If he wishes, he can refuse to accept the grant.)

(a) True or False? If good 1 is a normal good, then the effect of the grant on his consumption must be the same as the effect of an unconstrained lump sum grant of an equal amount. If this is true, prove it. If this is false, prove that it is false.

(b) True or False? If good 1 is an inferior good for the above consumer at all incomes $m > x_1^* + x_2^*$, then if he is given a grant of g_1 which must be spent on good 1, the effect must be the same as an unconstrained grant of an equal amount. If this is true, prove it. If this is false, show what he will do if he is given the grant.

(c) Suppose that the consumer discussed above has homothetic preferences and is currently consuming $x_1^* = 12$ and $x_2^* = 36$. Draw a graph with g_1 on the horizontal axis and the amount of good 1 on the vertical axis. Use this graph to show the amount of good 1 that the consumer will demand if his ordinary income is $m = 48$ and if he is given a grant of g_1 which must be spent on good 1. At what level of g_1 will this graph have a kink? (Think for a minute before you answer this. Give a numerical answer.)