Practice Problems #1

Interpret the following equations where C is the cost, and Q is quantity produced by the firm

a) $C(Q) = 10 + 2Q$

Costs depend on quantity. If the firm produces nothing, costs are 10, i.e. $C(0)=10+2(0)=0$. The 10 represents fixed costs. This is a short run cost function. For every additional unit of production, cost increase by 2, i.e. $d\,C(Q)/d\,Q=2$, i.e. the marginal cost is 2.

b) $C(Q) = 10Q$

Costs depend on quantity. If the firm produces nothing, costs are 0, i.e. $C(0)=10(0)=10$. The fact that there are no fixed cost implies a long-run cost function. For every additional unit of production, cost increase by 10, i.e. $d\,C(Q)/d\,Q=10$, i.e. the marginal cost is 10.

c) $C(Q) = 10 + Q^2$

Costs depend on quantity. If the firm produces nothing, costs are 10, i.e. $C(0)=10+(0)^2 =10$. The 10 represents fixed costs. This is a short run cost function. For every additional unit of production, cost increase by 2Q, i.e. $d\,C(Q)/d\,Q=2Q$, i.e. the marginal cost is 2Q. This means that the cost of producing an extra unit of output depends on the initial level of production. If you are already producing on a large scale, say Q=100, then producing an additional unit of production will incur an additional cost of 200, i.e. 2(100).

d) $C(Q) = 4 + 2Q^2$

Costs depend on quantity. If the firm produces nothing, costs are 4, i.e. $C(0)=4+2(0)^2 =4$. The 4 represents fixed costs. This is a short run cost function. For ever additional unit of production, cost increase by 4Q, i.e. $d\,C(Q)/d\,Q=4Q$, i.e. the marginal cost is 4Q. This means that the cost of producing an extra unit of output depends on the initial level of production. If you are already producing on a large scale, say Q=1000, then producing an additional unit of production will incur an additional cost of 4000, i.e. 4(1000).

Practice Problems #2

Evaluate the cost functions when Q=2 for each problem in Practice Problems #1

a) $C(2) = 10 + 2(2)=14$. It costs 14 to produce 2 units.

b) $C(2) = 10(2)=20$. It costs 20 to produce 2 units.
c) $C(2) = 10 + 2^2=14$. It costs 14 to produce 2 units.
d) $C(2) = 4 + 2(2)^2=12$. It costs 12 to produce 2 units.

Practice Problems #3

Interpret the following equations where C is the cost, Q is quantity produced by the firm and, a and b are known constants

a) $C(Q) = a + Q$

Costs depend on quantity. If the firm produces nothing, costs are a, i.e. $C(0)=a+(0)=a$. The a represents fixed costs. This is a short run cost function. "a" must therefore be a positive constant. For ever additional unit of production, cost increase by 1, i.e. $d\,C(Q)/d\,Q=1$, i.e. the marginal cost is 1.

b) $C(Q) = 10 + bQ$

Costs depend on quantity. If the firm produces nothing, costs are 10, i.e. $C(0)=10+b(0)=10$. The 10 represents fixed costs. This is a short run cost function. For ever additional unit of production, cost increase by b, i.e. $d\,C(Q)/d\,Q=b$, i.e. the marginal cost is b. One should think that "b" should be positive, producing an additional unit of output should incur additional costs.

c) $C(Q) = a + bQ$

Costs depend on quantity. If the firm produces nothing, costs are "a", i.e. $C(0)=a+b(0)=a$. The a represents fixed costs. This is a short run cost function. For ever additional unit of production, cost increase by b, i.e. $d\,C(Q)/d\,Q=b$, i.e. the marginal cost is b. One should think that "b" should be positive, producing an additional unit of output should incur additional costs.

1. Manipulation of an Equation

i) Adding, subtracting, multiplying, dividing (while still preserving the balance)

Practice Problems #4

Solve for P

a) $Q_D = 10 – P$

$$
\begin{aligned}
Q_D &= 10 - P \\
Q_D + P &= 10 - P + P && \text{(adding P to both sides)} \\
Q_D + P &= 10 \\
Q_D + P - Q_D &= 10 - Q_D && \text{(subtracting } Q_D \text{ from both sides)} \\
P &= 10 - Q_D
\end{aligned}
$$

b) $Q_D = 10 – 10P$

$$
\begin{aligned}
Q_D &= 10 - 10P \\
Q_D + 10P &= 10 - 10P + 10P && \text{(adding 10P to both sides)} \\
Q_D + 10P &= 10 \\
Q_D + 10P - Q_D &= 10 - Q_D && \text{(subtracting from } Q_D \text{ both sides)} \\
10P &= 10 - Q_D \\
(10P)/10 &= (10 - Q_D)/10 && \text{(dividing both sides by 10)} \\
P &= (10 - Q_D)/10
\end{aligned}
$$

c) $Q_S = 10 + P$

$$
\begin{aligned}
Q_S &= 10 + P \\
Q_S - 10 &= 10 + P - 10 \\
Q_S - 10 &= P
\end{aligned}
$$

d) $Q_S = 10 +bP$ where b is a known constant

$$
\begin{aligned}
Q_S &= 10 + bP \\
Q_S - 10 &= 10 + bP - 10 && \text{(subtracting 10 from both sides)} \\
Q_S - 10 &= bP \\
(Q_S - 10)/b &= (bP)/b && \text{(dividing both sides by b)} \\
(Q_S - 10)/b &= P
\end{aligned}
$$

e) $Q_D = a-bP$ where a and b are known constants

Q_D	$= a-bP$	
$Q_D + bP$	$= a-bP + bP$	(adding bP to both sides)
$Q_D + bP$	$= a$	
$Q_D + bP - Q_D$	$= a - Q_D$	(subtracting Q_D from both sides)
bP	$= a - Q_D$	
$(bP)/b$	$= (a - Q_D)/b$	(dividing both sides by b)
P	$= (a - Q_D)/b$	

ii) Cross multiplying: If x/y=w/z, then xz=yw

Practice Problems #5
Solve for y

a) $y/x=x/2$

$2y$	$=x^2$	(cross multiplying)
$(2y)/2$	$=(x^2)/2$	(dividing by 2)
y	$=(x^2)/2$	

b) $x/y=y$

x/y	$=y$	
x	$=y^2$	(cross multiplying)
$(x)^{0.5}$	$=(y^2)^{0.5}$	(taking the square root of both sides)
$(x)^{0.5}$	$=y$	

c) $(x+2)/(y+3) = 10/4$

$(x+2)/(y+3)$	$= 10/4$	
$4(x+2)$	$=10(y+3)$	(cross multiplying)
$4x+8$	$=10y+30$	
$4x+8-30$	$=10y+30-30$	(subtracting 30 from both sides)
$4x-24$	$=10y$	
$(4x-24)/10$	$= (10y)/10$	(dividing by 10 on both sides)
$(4x-24)/10$	$=y$	

Solving Two Equations and Two Unknowns

Approach 1: Substituting one equation into the other

Approach 2: Modifying one equation and adding or subtracting one equation from the other

Practice Problems #6

Use Approach 1 to solve the following problems

a) $x - 2y = 8$ (1)
$3x + y = 3$ (2)

Solve for x in equation (1)

$x - 2y = 8$
$x - 2y + 2y = 8 + 2y$ (adding 2y to both sides)
$x = 8 + 2y$

substitute the above equation into equation (2)

$3x + y = 3$
$3(8 + 2y) + y = 3$ (substituting for x)
$24 + 6y + y = 3$ (algebra)
$24 + 7y = 3$
$24 + 7y - 24 = 3 - 24$ (subtracting 24 from both sides)
$7y = 3 - 24$
$7y = -21$
$(7y)/7 = (-21)/7$ (dividing both sides by 7)
$y = -3$

substitute the above equation into equation (2)

$3x + y = 3$
$3x - 3 = 3$ (substituting for y)
$3x - 3 + 3 = 3 + 3$ (subtracting 3 from both sides)
$3x = 6$
$(3x)/3 = (6)/3$ (dividing both sides by 3)
$x = 2$

b) $3x + 3y = 4$ (1)
$x - y = 10$ (2)

Solve for x in equation (2)

$x - y = 10$
$x - y + y = 10 + y$

$x = 10+y$

substitute the above equation into equation (1)

$$
\begin{aligned}
3x + 3y &= 4 \\
3(10+y) + 3y &= 4 \\
30 + 3y + 3y &= 4 \\
30 + 6y &= 4 \\
30 + 6y - 30 &= 4 - 30 \\
6y &= -26 \\
(6y)/6 &= (-26)/6 \\
y &= -13/3
\end{aligned}
$$

substitute the above into equation (1)

$$
\begin{aligned}
3x + 3y &= 4 \\
3x + 3(-13/3) &= 4 \\
3x + -13 &= 4 \\
3x + -13 +13 &= 4 +13 \\
3x &= 17 \\
(3x)/3 &= 17/3 \\
x &= 17/3
\end{aligned}
$$

c) $P = 10 - Q$ (1)

$Q = 2 + 2P$ (2)

Substitute equation (2) into equation (1)

$$
\begin{aligned}
P &= 10 - (2 + 2P) \\
P &= 10 - 2 - 2P \\
P &= 8 - 2P \\
3P &= 8 \\
P &= 8/3
\end{aligned}
$$

Substitute into equation (2)

$Q = 2 + 2P$

$Q = 2 + 2(8/3)$

$Q = 22/3$

Practice Problems #7

Solve the problems in Practice Problem #6 using Approach 2

a) $x - 2y = 8$ (1)
$3x + y = 3$ (2)

Multiply equation (2), and add equation (1)

$$\begin{array}{l} x - 2y = 8 \\ \underline{6x + 2y = 6} \\ 7x = 14 \end{array}$$

$x=2$

substitute into equation (2)

$$\begin{array}{ll} 3(2) + y & = 3 \\ 6 + y & = 3 \\ y & =-3 \end{array}$$

b) $3x + 3y = 4$ (1)
$x - y = 10$ (2)

multiply equation (2) by 3 and add equation (1)

$$\begin{array}{ll} 3x - 3y & = 30 \\ \underline{3x + 3y} & \underline{= 4} \\ 6x & =34 \\ x & =17/3 \end{array}$$

c) $P = 10 - Q$ (1)
$Q = 2 + 2P$ (2)

Multiplying equation (1) by 2 and rewriting it, and adding equation (2)

$$\begin{array}{ll} 2Q & = 20 - 2P \\ \underline{Q} & \underline{= 2 + 2P} \\ 3Q & =22 \\ Q & =22/3 \end{array}$$

$P=10-Q$
$P=10 - 22/3$
$P=8/3$

Afternoon Session

Practice Problems #1

Evaluate the following production function: $f(L,K) = LK^2$ when

a) L=2 and K=1

$f(2,1) = 2(1)^2=2$ i.e. output is 2 when L is 2 and K is 1.

b) L=3 and K=1

$f(3,1) = 3(1)^2=3$ i.e. output is 3 when L is 3 and K is 1.

c) L=4 and K=1

$f(4,1) = 4(1)^2=4$ i.e. output is 4 when L is 4 and K is 1.

d) L=2 and K=2

$f(2,2) = 2(2)^2=8$ i.e. output is 8 when L is 2 and K is 2

Note: The productivity of an extra unit of labour depends on the capital stock

e) L=3 and K=2

$f(3,2) = 3(2)^2=12$ i.e. output is 12 when L is 3 and K is 2

Note: The productivity of an extra unit of labour depends on the capital stock

Practice Problems #2

Evaluate the following utility function: $U(x_1, x_2) = 2x_1 + 4x_2$ when

a) $x_1= 1$ $x_2=1$

$U(1,1) = 2(1) + 4(1)=6$

b) $x_1= 1$ $x_2=2$

$U(1,2) = 2(1) + 4(2)=10$

c) $x_1= 2$ $x_2=1$

$U(2,1) = 2(2) + 4(1)=8$

d) $x_1= 2$ $x_2=2$

$U(2,2) = 2(2) + 4(2)=12$

Note: The extra satisfaction of extra unit of x_1 does not depend on the consumption of x_2 (i.e. it is always 4)

2. Partial Derivatives

How will the function change if one changes only one variable (holding the other constant)

Example

How will a change in x_1 affect the utility of the consumer for each of the following (i.e. $dU(x_1, x_2)/dx_1$)

a) $U(x_1, x_2) = 2x_1 + 4x_2$

b) $U(x_1, x_2) = x_1^2 x_2$

Practice Problems #3

Calculate the impact of a change in labour on output for each of the following production functions

a) $f(L,K) = 2L + 3K$

$df(L,K)/dL = 2$ holding the amount of capital fixed, an increase in L of one unit increases output by 2 units.

b) $f(L,K) = 2LK$

$df(L,K)/dL = 2K$ holding the amount of capital fixed, an increase in L of one unit increases output by 2K units. So if the firm had 4 units of capital, the extra worker would add 2(4)=8 units of output.

c) $f(L,K) = 2L + 3LK$

$df(L,K)/dL = 2 + 3K$ holding the amount of capital fixed, an increase in L of one unit increases output by 2+3K units. So if the firm had 4 units of capital, the extra worker would add 2+3(4)=14 units of output.

d) $f(L,K) = 2L^2K$

$df(L,K)/dL = 4LK$ holding the amount of capital fixed, an increase in L of one unit increases output by 4LK units. So if the firm had 4 units of capital, and 4 units of labour, the extra worker would add 4(4)(4)=64 units of output.

e) $f(L,K) = 2L^{1/2}K^{1/2}$

$df(L,K)/dL = L^{-1/2}K^{1/2}$

f) $f(L,K) = AL^{\alpha} K^{1-\alpha}$

$$df(L,K)/dL = \alpha A L^{\alpha-1} K^{1-\alpha}$$

3. Optimization

a) Unconstrained maximization – One variable

Practice Problems #4
Calculate the profit maximizing output for each of the following revenue and cost functions (using the "maximization" approach)
a) $R(Q)=20Q$ and $C(Q)= 2Q + Q^2$

Max_Q Profits
$\text{Max}_Q\ R(Q) – C(Q)$
$\text{Max}_Q\ 20Q – 2Q+Q^2$

$20 – 2+2Q = 0$
$Q^* = 9$

b) $R(Q)=aQ$ and $C(Q)= 2Q + Q^2$ where a is a known constant

Max_Q Profits
$\text{Max}_Q\ R(Q) – C(Q)$
$\text{Max}_Q\ aQ – 2Q+Q^2$

$a – 2+2Q = 0$
$Q^* = (a-2)/9$

b) Unconstrained maximization – Two variables

Example: Maximizing utility by choosing the consumption of two goods (i.e. x_1 and x_2) where $U(x_1, x_2) = 10x_1 - x_1^2+ 4x_2 – x_2^2$

$\text{Max}_{x1,x2}\ U(x_1, x_2)$
$\text{Max}_{x1,x2}\ 10x_1 - x_1^2+ 4x_2 – x_2^2$

$10 – 2x_1 = 0$ (derivative with respect to x_1 and equating to zero)
$4 – 2x_2 = 0$ (derivative with respect to x_2 and equating to zero)

(2 equations, 2 unknowns)

Practice Problems #6
Calculate the utility maximizing level of consumption (of the two goods) for each of the following utility functions
a) $U(x_1, x_2) = 10 + 20x_1 - 2x_1^2 + 4x_2 – x_2^2$

$\text{Max}_{x1,x2}\; U(x_1, x_2)$
$\text{Max}_{x1,x2}\; 10x_1 + 20x_1 - 2x_1^2 + 4x_2 – x_2^2$

$20 – 4x_1 = 0$ (derivative with respect to x_1 and equating to zero)
$4 – 2x_2 = 0$ (derivative with respect to x_2 and equating to zero)

$x_1^* = 5$
$x_2^* = 2$

b) $U(x_1, x_2) = 10x_1 - x_1^2 + 40x_2 – 10x_2^2$

$\text{Max}_{x1,x2}\; U(x_1, x_2)$
$\text{Max}_{x1,x2}\; 10x_1 - x_1^2 + 40x_2 – 10x_2^2$

$10 – 2x_1 = 0$ (derivative with respect to x_1 and equating to zero)
$40 – 20x_2 = 0$ (derivative with respect to x_2 and equating to zero)

$x_1^* = 5$
$x_2^* = 2$

c) Constrained maximization – Two variables

Practice Problems #7
Calculate the utility maximizing level of consumption (of the two goods) for each of the following utility functions, price, and income levels
a) $U(x_1, x_2) = x_1x_2$, $p_1 = 2$, $p_2 = 2$, $y=200$

$\text{Max}_{x1,x2}\; U(x_1, x_2)$ s.t. $P_1x_1 + P_2x_2 = y$

$\text{Max}_{x1,x2}\ x_1x_2$ s.t. $2x_1 + 2x_2 = 200$

$\text{Max}_{x1,x2}\ x_1x_2$ s.t. $x_1 = 100 - x_2$

$\text{Max}_{x2}\ (100 - x_2)x_2$

$\text{Max}_{x2}\ 100\, x_2 - x_2^2$

$100 - 2x_2 = 0$

$x_2^* = 50$

$x_1 = 100 - x_2$
$x_1 = 100 – 50$
$x_1^* = 50$

b) $U(x_1, x_2) = x_1x_2^2$, $p_1 = 1$, $p_2 = 1$, $y=00$

$\text{Max}_{x1,x2}\ U(x_1, x_2)$ s.t. $P_1x_1 + P_2x_2 = y$

$\text{Max}_{x1,x2}\ x_1x_2^2$ s.t. $2x_1 + 2x_2 = 200$

$\text{Max}_{x1,x2}\ x_1\, x_2^2$ s.t. $x_1 = 100 - x_2$

$\text{Max}_{x2}\ (100 - x_2)\, x_2^2$

$\text{Max}_{x2}\ 100\, x_2^2 - x_2^3$

$200\, x_2 - 3\, x_2^2 = 0$
$200 - 3\, x_2 = 0$
$x_2 = 200/3$

$x_1 = 100 - x_2$
$x_1 = 100 – 200/3$
$x_1^* = 100/3$