Formal semantics

✓Predicate logic-logical relations within the sentence

Propositional logic-logical relations between sentences

Metalanguage

Truth-conditional semantics and the Fregean Program

There is a bag in my car.-

Alfred Tarski: the meaning of a sentence is to know its truth-conditions

What must be the world like so that we can say if the sentence is true or not? As long as there is a bag in my car, the sentence is true. {0,1}

Is the meaning of the sentence only its truth-conditions?

- Compositional semantics: Frege, Montague

- More interested in the semantic composition

of sentences:

Frege:

“sentences are split up into two parts: one complete in itself and the other in need of supplementation, or ‘unsaturated’… Only when this place [the unsaturated one] is filled up with a proper name, or with an expression…, does a complete sense appear.”

Using the metalanguage

- Bruce is clever.
- Bruce sleeps.
- A property is predicated of an individual.

- b- individual constant
- C, S - predicates (predicate constants)
- S(b); C(b)

Formalizing sentences

C(b) -proposition(predicate and individual constant)

C(x)-open sentence(predicate and individual variable)

x is not an individual constant but an individual variable. It does not stand for any particular individual but for <u>any indivudual.</u>

<u>X stands for an arbitrary individual</u>

F(x) - predicate variable and individual variable

A predicate variable stands for <u>any property</u>

Individual term

- Constants and variables are called terms. Individual terms following the predicate are called arguments.
- The sentences below have the same logical structure: they consist of one predicate term and one individual term
- Tarzan is asleep. A(t)
- Tarzan surrendered. S(t)
- Tarzan is brave. B(t)
- Tarzan is a man. M(t)

P(t)

P-predicate term

t-Tarzan (individual term), argument

Sentences with more than one term

- Tarzan found a hammer. F(t,h)
- Tarzan admires Jane. A(t,j)
- Tarzan loves the monkeys.L(t,m)
- Tarzan is wiser than Thor.W(t,th)

Here the predicate takes two arguments (individual terms). This type of predicate is called <u>two-place predicate</u>. Predicates that take only
one argument are called <u>one-place predicate</u>.

Predicate structure

$P(t)$ one-place

$P(t_1,t_2)$two-place

$P(t_1,t_2,t_3)$three-place

$P(t_1,t_2,\ldots t_n)$ n-place

Tarzan gave Jane the apple.

$G(t,j,a)$

Quantifiers
Universal Quantifier $\forall$

- All students are clever.
- C(all students)

Problem with this representation: the expression 'all students' does not refer to an individual the same way as names do. We need another one.

What this sentence says is that if we find a student we expect it to be clever, or if somebody is a student, he is clever.

Universal quantifier $\forall$: logical constant with the meaning every/all

$\forall x\ (S(x) \longrightarrow C(x))$

How to read a statement with an universal quantifier

$\forall(x)(S(x) \longrightarrow C(x))$

x-any individual in a domain/universe, does not refer to any particular individual.

- ∀ is a logical operator that **ranges over all individuals in a domain/universe.**
- For every x it holds that if x is a student, x is clever.
- For everything, it holds that if it is a student, it is clever
- If something is a student, then it is clever
- All students are clever/Every student is clever.

A less complex open proposition:

Everybody is clever.

C(x): a property C applies to an individual, which is a variable (not a particular one) but in which way?

How exactly do we represent this statement?

$\forall$xC(x)-everybody has the property of being clever.

The property of being clever holds for everybody.

- C(x) - x, x being any individual in a domain, has a property C.
- **Domain** {Bill, Anna, Tom}
- x=Bill
- x=Anna
- x=Tom

Steve? He does not belong to the domain, therefore he will not satisfy the property.

What is x, why not &?

$\forall(x)(S(x) \longrightarrow C(x))$

The logical connective is an implication.

Why don't we use the logical connective & (conjunction?

If we use &, we would state that every individual is both clever and student, which is not the same.

x is the same in both expressions. We say that the quantifier binds the x's.

A variable, bound by a quantifier is called a bound variable.

The parenthesis after the quantifier shows its SCOPE.

Existential quantifier -∃

There is an x, such that x is a student.

The existentail quantifier states that there is at least one member in the universe of discourse that satisfies the open sentence S(x), e.g.

$\exists x S(x)$ - formula

Natural language sentences corresponding to the formula:

Some student/at least one student/a student

There exists a student.

Someone is a student. A student exists.