MAT1332 Spring/Summer 2010

Assignment 4, Due July 15, 2010.

Give decimal expansions to four places. You must justify your answers to all of the questions below.

1. Find the eigenvalues and eigenvectors of the following matrix:

$$\begin{bmatrix} 4 & 0 & 3 \\ 2 & 5 & 0 \\ 0 & 2 & 0 \end{bmatrix}$$

2. Find the eigenvalues and eigenvectors of the following matrix:

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & -6 \\ -1 & 1 & 0 \end{bmatrix}$$

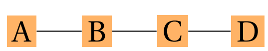
3. Suppose we study an ant colony in which there is no biological specialization of the ants for different jobs (i.e., the same ant could at one time be a "soldier" and at another time be a "worker"). Assume also that the total population of the colony remains fixed for the duration of the study. Ants can be "employed" at any given time as either soldiers (S) or workers (W). We observe that 50% of soldiers will switch to being workers at any given time. On the other hand, 20% of workers will switch to being soldiers.

   (a) Write the transition matrix A describing this dynamical system.

   (b) Find the eigenvalues and eigenvectors of A.

   (c) Find the equilibrium vector v_* of A. (I.e., v_* should be a column vector $\begin{bmatrix} s \\ w \end{bmatrix}$ such that $s + w = 1$ and $0 \leq s, w \leq 1$ and such that $Av_* = v_*$.)

4. A test subject is allowed to freely roam between four rooms (A, B, C and D) which are connected to one another in the following way:



(i.e., room A is connected to room B but not to any of the other rooms; room C is connected to rooms B and D but not to room A). After observing the test subject we note that he behaves each minute according to the following rules:

- If he is is in room A, then the probability that he will remain in room A after one minute is $\frac{1}{2}$.

- If he is in room B, then he will move after one minute to room A with probability $\frac{1}{3}$ and to room C with probability $\frac{1}{2}$.
- If he is in room C, then he will move after one minute to room B with probability $\frac{1}{4}$ and to room D with probability $\frac{1}{4}$.
- If he is in room D, then he will move after one minute to room C with probability $\frac{5}{6}$.

Assume that the subject is only able to move each minute between adjacent rooms (i.e., he cannot move from room A to room C in one minute).

Write a transition matrix describing this system. If the subject starts (at time 0) in room B, what is the probability that after one hour he is in (give your answer to two decimal places so that rather than, say, 15% you should have 15.45%, or what have you):

(a) room A,

(b) room B,

(c) room C, or

(d) room D?

Hint: Remember to calculate the eigenvalues and eigenvectors of the transition matrix and then to write the initial condition vector as a linear combination of the eigenvectors.