

MAT1332 Spring/Summer 2010

Assignment 3, Due June 22, 2010.

Give decimal expansions to four places. You must justify your answers to all of the questions below.

1. Solve the differential equation

$$\frac{dv}{dt} = e^{-2t}v^3$$

where v is a positive function with the initial condition $v(0) = \frac{1}{2}$. **Solution:** $v =$

$$(3 + e^{-2t})^{-\frac{1}{2}}$$

2. Consider the equation

$$f'(x) = e - e^{f(x)}.$$

- (a) Graph the phase line diagram and the direction field in the interval $-3 \leq x \leq 3$.
- (b) Calculate the equilibrium point(s) and determine their stability using a calculation. Compare this with the graphical result obtained in part (a).

Solution: There is a typo in the statement of this problem it should read $f'(x) = e - e^x$ instead of $f'(x) = e - e^{f(x)}$.

3. Consider the complex numbers $z = 4 - 2i$ and $w = -2 + 3i$.

- (a) Calculate $z + w$, zw , z/w and w/z .
- (b) Show that z is a solution of the equation $u^2 - 8u + 20 = 0$ (in the unknown u).
- (c) Solve the equation $u^2 + 4u + 13 = 0$ (in the unknown u).

Solution:

- (a) $2 + i, -2 + 16i, -\frac{14}{13} - \frac{8}{13}i, -\frac{7}{10} + \frac{2}{5}i$.
- (b) This is a direct calculation.
- (c) $2 \pm 3i$.

4. Do the following:

- (a) Find the modulus and argument of the following complex numbers: $u = \sqrt{2} + \sqrt{2}i$, $v = 3 + 3\sqrt{3}i$ and $w = 2\sqrt{3} + 2i$.
- (b) Write the complex numbers $u = 9e^{3i}$, $v = -3e^{-\frac{\pi}{5}i}$ and $w = 3e^{20i}$ in the form $x + iy$.
- (c) Find $\bar{u}v$ where u, v are as in part (a).
- (d) Find w^{-1} where w is as in part (a).

Solution:

- (a) $|u| = 2$, $\arg(u) = \arctan(1) = \frac{\pi}{4}$, $|v| = 6$, $\arg(v) = \arctan(\sqrt{3}) = \frac{\pi}{3}$, $|w| = 4$ and $\arg(w) = \arctan(\frac{1}{\sqrt{3}}) = \frac{\pi}{6}$.
- (b) $u = 9 \cos(3) + 9i \sin(3)$, $v = -3 \cos(\frac{\pi}{5}) - 3i \sin(\frac{\pi}{5})$, $w = 3 \cos(20) + 3i \sin(20)$.
- (c) $3\sqrt{2} + 3\sqrt{6} + (3\sqrt{6} - 3\sqrt{2})i$.
- (d) $\frac{\sqrt{3}}{8} - \frac{1}{8}i$.