

MAT1332 Spring/Summer 2010

Assignment 3, **Due June 22, 2010.**

Give decimal expansions to four places. You must justify your answers to all of the questions below.

1. Solve the differential equation

$$\frac{dv}{dt} = e^{-2t}v^3$$

where v is a positive function with the initial condition $v(0) = \frac{1}{2}$.

2. Consider the equation

$$f'(x) = e - e^{f(x)}.$$

- (a) Graph the phase line diagram and the direction field in the interval $-3 \leq x \leq 3$.
 - (b) Calculate the equilibrium point(s) and determine their stability using a calculation. Compare this with the graphical result obtained in part (a).
3. Consider the complex numbers $z = 4 - 2i$ and $w = -2 + 3i$.
 - (a) Calculate $z + w$, zw , z/w and w/z .
 - (b) Show that z is a solution of the equation $u^2 - 8u + 20 = 0$ (in the unknown u).
 - (c) Solve the equation $u^2 + 4u + 13 = 0$ (in the unknown u).

4. Do the following:

- (a) Find the modulus and argument of the following complex numbers: $u = \sqrt{2} + \sqrt{2}i$, $v = 3 + 3\sqrt{3}i$ and $w = 2\sqrt{3} + 2i$.
- (b) Write the complex numbers $u = 9e^{3i}$, $v = -3e^{-\frac{\pi}{5}i}$ and $w = 3e^{20i}$ in the form $x + iy$.
- (c) Find $\bar{u}v$ where u, v are as in part (a).
- (d) Find w^{-1} where w is as in part (a).