

**MAT1332 Spring/Summer 2010**

**Assignment 2, Due June 3, 2010.**

Give decimal expansions to four places. You must justify your answers to all of the questions below.

1. Suppose the quantity  $\nu$  of a radioactive isotope decays according to the law

$$\frac{d\nu}{dt} = -k\nu \quad (1)$$

with  $k > 0$  a constant,  $\nu$  measured in moles and  $t$  measured in years. Assume that at time  $t = 0$  there are 100 moles of the isotope and at  $t = 2$  there are 90 moles remaining.

- Solve the differential equation (1), with the initial condition given above, using separation of variables.
  - Find the value of  $k$  using your solution to (a).
  - Determine the half-life of the isotope using your solutions to (a) and (b). (The half-life is the time  $t$  at which  $\nu(t) = \frac{1}{2}\nu(0)$ .)
  - How much of the isotope remains after 20 years?
2. Do the following integrals converge? (If so, then give their values and if not then prove it.)

(a)  $\int_1^{\infty} \frac{x}{e^x} dx$ .

(b)  $\int_{\frac{1}{2}}^1 \frac{1}{t^2\sqrt{1-t^2}} dt$ .

(c)  $\int_2^{\infty} \frac{5}{2x^2+4x-12} dx$ .

(d)  $\int_0^1 \frac{1}{xe^x} dx$ .

3. To a barrel holding 500l of water is added 25kg of salt. A hole is made in the barrel so that 50l of water flows out of the barrel per minute. Simultaneously 50l per minute of fresh water is added to the barrel. Assume that the contents of the barrel are continuously stirred to ensure a uniform mixture.
- Write the autonomous differential equation describing the rate of change (in kilograms per minute) of the quantity  $\nu$  of salt in the barrel.
  - Solve the differential equation from (a) using separation of variables.
  - How long before only 5kg of salt remains in the barrel?