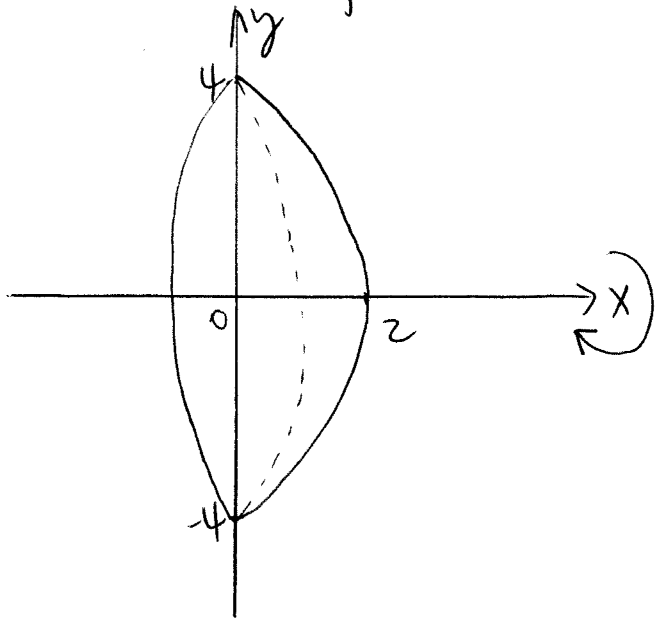
The solutions to the extra problems of DGD week 3.

Integrals and Volumes:

1. For $y=4-x^2$, $y=0$, $x=0$ (in the first quadrant).

Solution: the graph is



$$V=\int_0^2 \pi [f(x)]^2 dx = \int_0^2 \pi (4-x^2)^2 dx$$

$$=\int_0^2 \pi(16-8x^2+x^4)dx$$

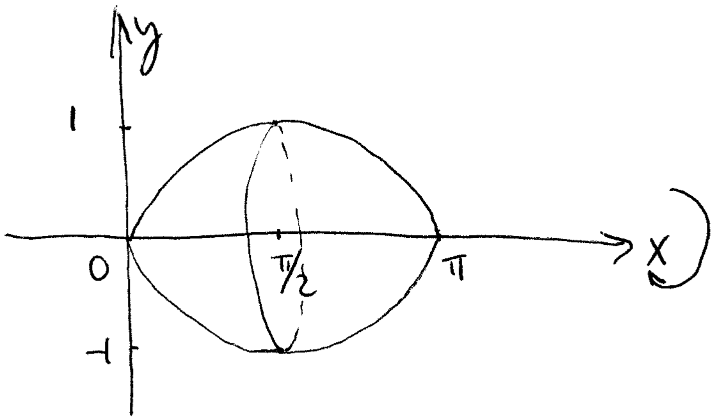
$$=\left(16\pi x - \frac{8\pi}{3}x^3 + \frac{\pi}{5}x^5\right)\Big|_0^2$$

$$=32\pi - \frac{64\pi}{3} + \frac{32\pi}{5}$$

$$=\frac{256}{15}\pi$$

2. For $y=\sqrt{\sin x}$, $0\le x\le \pi$, $y=0$.

Solution:



$$V=\int_0^\pi \pi[f(x)]^2 dx$$

$$=\int_0^\pi \pi[\sqrt{\sin x}]^2 dx$$

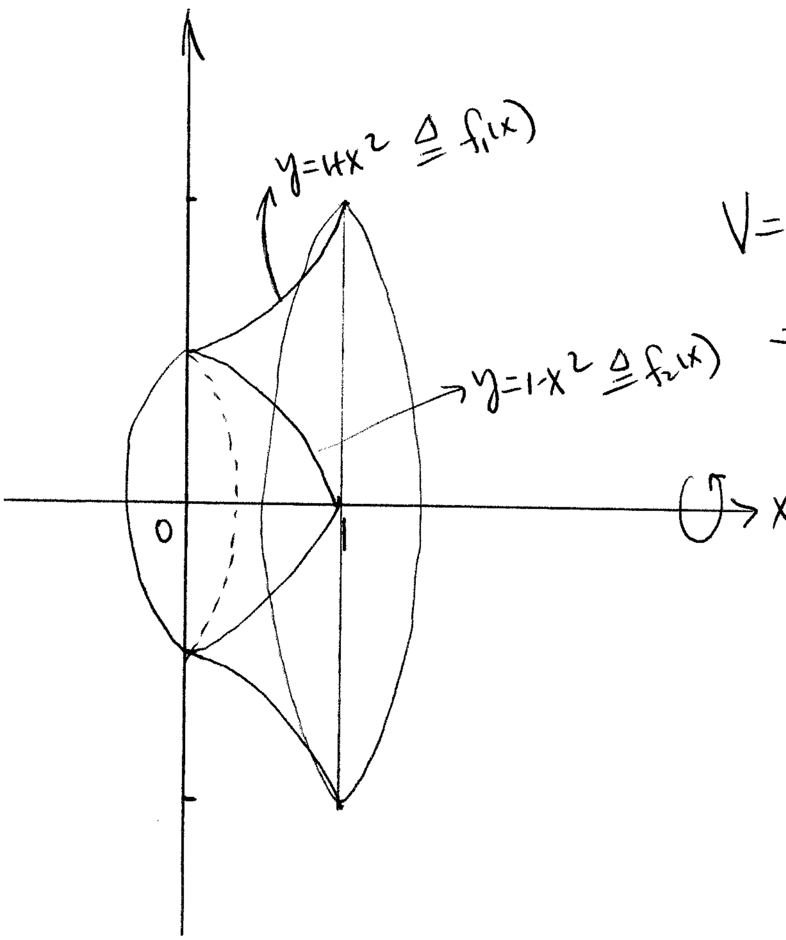
$$=\int_0^\pi \pi \sin x\, dx$$

$$=\pi(-\cos x)\Big|_0^\pi$$

$$=2\pi$$

3. For $y=1-x^2$, $y=1+x^2$, $0 \le x \le 1$.

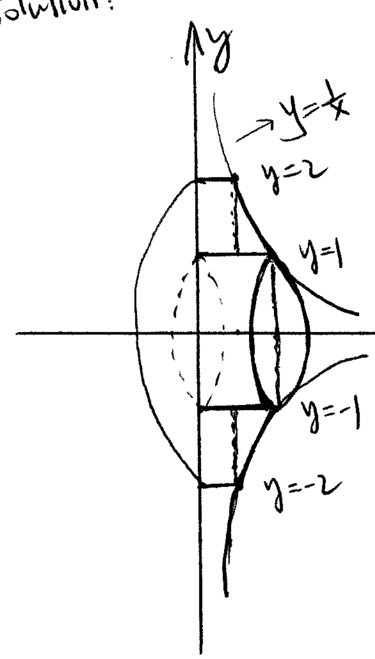
Solution.



$$V=\int_0^1 \pi [f_1(x)]^2 dx - \int_0^1 \pi [f_2(x)]^2 dx$$
$$=\int_0^1 \pi (1+x^2)^2 dx - \int_0^1 \pi (1-x^2)^2 dx$$
$$=\int_0^1 \pi (1+2x^2+x^4) dx - \int_0^1 \pi (1-2x^2+x^4) dx$$
$$=\int_0^1 4\pi x^2 dx$$
$$=\frac{4\pi}{3} x^3 \Big|_0^1$$
$$=\frac{4\pi}{3}$$

4. For $y=\frac{1}{x}$, $x=0$, $y=1$, $y=2$ (in the first quadrant)

Solution:



when $y=1$, for $y=\frac{1}{x}$, we have $x=\frac{1}{y}=\frac{1}{1}=1$.

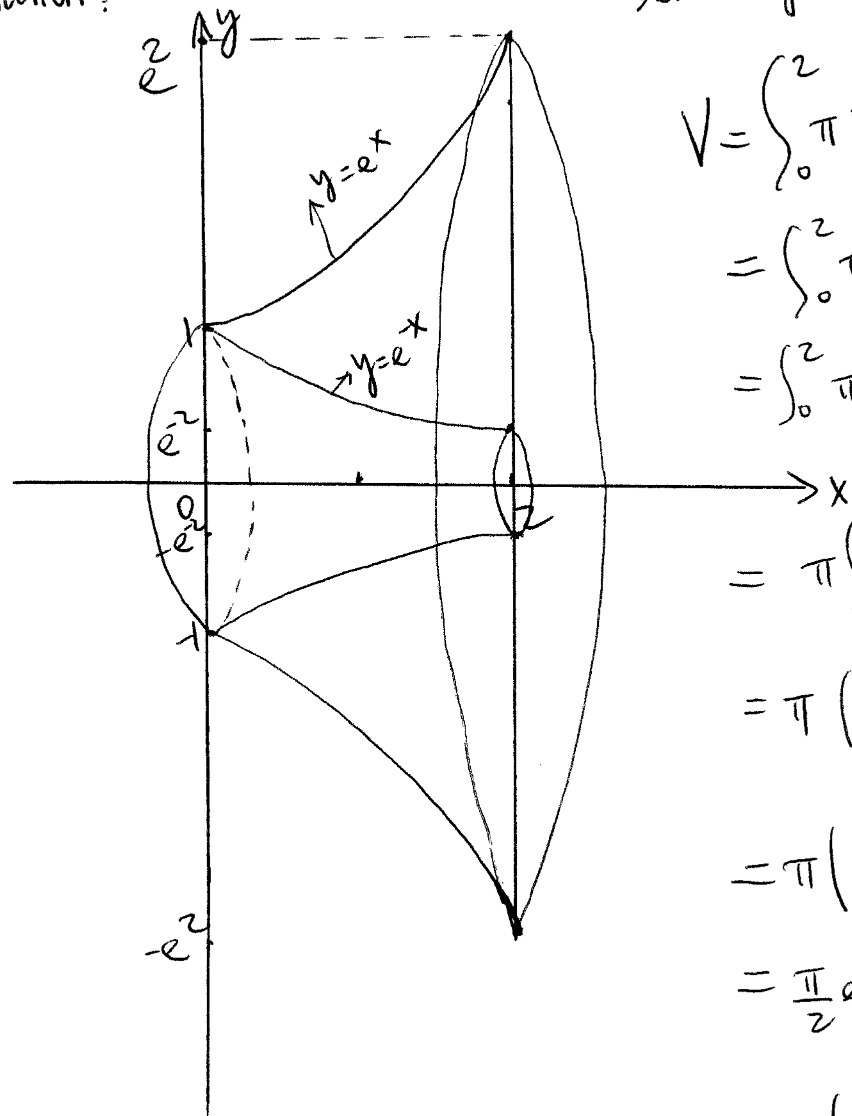
So $x \in [0,1]$.

$$V=\left(\int_0^1 \pi 2^2 dx - \int_0^1 \pi 1^2 dx\right) + \left(\int_0^1 \pi \left(\frac{1}{x}\right)^2 dx - \int_0^1 \pi 1^2 dx\right)$$
$$=2\pi \int_0^1 dx + \pi \int_0^1 \frac{1}{x^2} dx$$
$$=2\pi x \Big|_0^1 + \pi \left(-\frac{1}{x}\right)\Big|_0^1$$
$$=2\pi - \pi$$
$$=\pi$$

5. For $y=e^x$, $y=e^{-x}$, $0 \le x \le 2$

Solution:

Denote $y=e^x \triangleq f_1(x)$, $y=e^{-x} \triangleq f_2(x)$



$$V=\int_0^2 \pi [f_1(x)]^2 dx - \int_0^2 \pi [f_2(x)]^2 dx$$

$$=\int_0^2 \pi (e^x)^2 dx - \int_0^2 \pi (e^{-x})^2 dx$$

$$=\int_0^2 \pi e^{2x} dx - \int_0^2 \pi e^{-2x} dx$$

$$= \pi \int_0^2 (e^{2x} - e^{-2x}) dx$$

$$= \pi \left(\frac{1}{2} e^{2x} - \frac{1}{-2} e^{-2x} \right) \Big|_0^2$$

$$= \pi \left(\frac{1}{2} e^4 + \frac{1}{2} e^{-4} \right) - \pi \left(\frac{1}{2} e^0 + \frac{1}{2} e^0 \right)$$

$$= \frac{\pi}{2} e^4 + \frac{\pi}{2} e^{-4} - \pi$$

$$= \pi \left(\frac{e^4 + e^{-4}}{2} - 1 \right)$$