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Property Crime with Private Protection: A Market-for-Offenses Approach*

We propose a market-for-offenses model of property crime, which explicitly accounts for protection expenditures among heterogeneous individuals. The crime equilibrium is modelled as a free-access equilibrium in which the match between criminals and victims equates the average returns to crime. We borrow from the literature on the economics of conflicts in order to define an appropriation function that combines the efforts of criminals with the protection efforts of the victims. The supply and demand for crime are endogenized taking into account incentives to participate in criminal activities and individual protection decisions. The effects of changes in public enforcement, redistribution policies and economic development are analysed, as well as the distribution of the burden of crime among heterogeneous individuals.

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1 Introduction

In the literature on the economics of crime, one typically finds public enforcement, private protection, legitimate wage opportunities and income distribution as fundamental determinants of property crime.\(^1\) It is also commonly recognized that these factors interact in the general economy in order to produce the equilibrium level of crime. The precise way in which these interactions take place is thus crucial to our understanding of how, say, economic growth, redistribution or public enforcement affect the level of crime. Our analysis is a contribution in this direction.

In order to account for the interactions between the different determinants of crime, it is necessary to develop a “market-for-offenses” model of crime with heterogeneous individuals for which the supply of criminal activities and the level of private protection are derived from their maximizing behavior.\(^2\) We have not found any model that does this while accounting for private decisions to protect oneself. Yet, most authors would agree that private protection constitutes a crucial element. Regarding empirical evidence for the USA for instance, Levitt (1999) reports that “the home security business has grown at an annual rate of 10 percent over the last decade and is now a $14 billion a year business” (p. 91); Shavell (1991) mentions that “private expenditures on security from crime exceed public expenditures” (p. 123); while DiIulio (1996) points out that the high rates of criminal victimization in inner-city areas can be partly explained by the lack of victims’ financial resources to protect their homes (p. 11). The extensive surveys of the crime literature by Polinsky and Shavell (2000) and Witte and Witt (2001) both mention, indeed, the fact that accounting for private protection efforts constitutes an important dimension of the crime problem that has not been sufficiently examined yet.

We reckon that the reason why no such “market-for-offenses” model of crime had been developed so far may due to the difficulty of matching criminals with victims in a tractable and convincing fashion. Our approach leads to a reasonably simple, yet insightful, characterization of the crime equilibrium. It first makes use of Gordon’s (1954) main result on free access to

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\(^1\)As will become clear later, our analysis concentrates on the problem of crime aimed specifically at an individual’s property, such as burglary, robbery, car theft, picking pockets, etc. It does not apply to such crimes as financial or consumer fraud, tax evasion, or organized crime.

\(^2\)The expression “market-for-offenses” was borrowed from Ehrlich (1996).
natural resources by similarly assuming that in a crime equilibrium, it must be the case that the average payoff to crime is equalized across all victims. It then borrows from the literature on the economics of conflicts by defining an appropriation technology which transforms the respective predation and protection efforts of criminals and victims into respective gains and losses from crime.

In the model, the individual decision to participate in crime depends on its payoff, and so do the allocations of predation efforts by criminals across all the potential victims and the protection decisions taken by those same victims. Hence, our crime equilibrium is somewhat analogous to usual goods markets for it satisfies a supply for crime emanating from individual incentives to engage in crime and a “demand”, or tolerance, for crime emanating from individual incentives to be protected against it, while the equilibrium is determined by the crime payoff which equalizes supply and demand. People differ by their wealth, or income opportunities, and we assume that wealthier individuals face a larger opportunity cost of engaging into crime. Property crime gives the opportunity for a criminal to appropriate a share of another individual’s wealth. The return to each unit of time spent trying to appropriate from a particular victim will of course depend on how that victim protects itself. But it will also depend negatively on the total amount of time spent trying to appropriate from that same victim by all criminals. We also introduce an exogenous public enforcement effort.

Somewhat surprisingly, in our model, the victims are positively affected by the gross returns to crime in the economy. Indeed, since the returns to crime must be the same across all victims, from the standpoint of one victim, a globally higher payoff to crime makes his wealth relatively less attractive, and thus easier to protect. This observation contrasts slightly with the usual approach, in both the theoretical and empirical literature, which assumes that people are concerned about the crime rate. Another result is that the share of wealth lost to crime and to protection expenditures is the same for all individuals, regardless of their initial wealth. In this respect, the model enables us to tackle the issue of the distributive burden of crime while accounting not only for losses from theft, but also for the oft-neglected protection expenditures (Levitt, 1999, p. 88). A further issue is that of the relation between economic growth and crime which, in our analysis, turns

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3By gross payoff to crime we mean the returns to crime gross of the probability of being apprehended and punished.
out to have ambiguous effects on the crime rate and the total value of goods stolen, but unambiguously increases the gross payoff to crime and thus the welfare of all. This is true even though some people do not benefit directly in the growth process in the sense that their wealth does not change. Finally, we obtain ambiguous effects for inequality. We show, in fact, that what really matters for crime is not whether income is better distributed or not, but rather how the indifferent criminal, i.e. the one who is indifferent between committing crimes or staying honest, is affected by the distribution.

We pursue our analysis by considering the public policy implications of the model. We show that increased public enforcement has the expected effects. We explore the conditions under which a specific uniform-tax redistribution scheme will be effective in reducing crime. And finally, we compare the relative effectiveness of public enforcement and redistribution as means of controlling crime. We show, in particular, that if redistribution can effectively target a certain group of potential criminals, then some combination of public enforcement and redistribution will be optimal. But if redistribution schemes are not well targeted, then it may be efficient to make use of public enforcement alone.

Aside from these results, we believe that our analysis' main contribution may rest more on its potential as a framework for further empirical studies on crime which incorporates private protection decisions.

In the crime literature, we have found few market-for-offenses models that explicitly endogenize the returns to crime. An early one is that of Skogh and Stuart (1982) who show how public enforcement of property rights can improve the lot of all individuals. This model is close to ours in spirit, with the difference that it considers only homogeneous individuals and is thus not very useful in its policy implications. Furlong (1987) also concentrates on the issue of public enforcement by cleverly introducing a probabilistic function that matches patrolmen with criminals and considers homogeneous victims. Fender (1999) introduces heterogeneous criminals but still assumes homogeneous victims. Chiu and Madden (1998) propose a market model of burglary in which the equilibrium crime rate is dictated by house prices. Imrohoroglu et al. (2000) do allow for both heterogeneous criminals and victims in an ambitious attempt to calibrate a general-equilibrium model of crime and labor.

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4In a way that will become clear later, we refer to public enforcement as a general word for “negative incentives” or “sticks”, while redistribution is our general word for “positive incentives” or “carrots”.
to the U.S.A. economy. But except for Skogh and Stuart (1982), none of the above models account for private protection efforts. Shavell (1991) does analyze individual decisions to protect oneself. However, he concentrates on the issue of observable versus non-observable protection and assumes identical victims and thieves, an exogenous number of thieves, and does not include public enforcement of law. The model presented in the survey by Ehrlich (1996) is certainly the closest in spirit to ours. Indeed, Ehrlich puts much emphasis on the importance of the elasticities of demand and supply of crime, an element which does turn out to be crucial to our results. We nonetheless believe that by explicitly accounting for the individually optimal choice of private protection levels, our model fills an important gap in the existing literature. Indeed, it allows us to perform complete analytical comparative-static experiments that bring out the exact role played by all parameters of the model which should be useful in conducting future empirical analysis.

Our analysis is also related to the literature on conflicts in which general-equilibrium models of appropriation were also developed.\footnote{Note that the term “appropriation” is typically used instead of “crime” in this literature since in most cases, the state is not present and thus the act of taking from another’s belongings cannot be considered illegal, even though it is done against his or her will.} Grossman (1995) considers the issue of income redistribution as a means of reducing “extralegal appropriations” by workers. He assumes homogeneous potential criminals and accounts for neither public nor private protection. Skaperdas (1992) analyzes the strategic interactions between two individuals who can choose between productive and appropriative activities. Although he does consider heterogeneous individuals, his analysis is limited to just two individuals and does not consider public enforcement. Hirshleifer (1995) similarly considers the case of more than two individuals separately from the case of heterogeneous individuals. Finally, Grossman and Kim (1995) consider the choice between offensive and defensive expenditures among heterogeneous individuals but again, they do so with only two agents.

The paper is organized as follows: Section 2 presents a short survey of empirical findings on crime. Section 3 presents property crime as a common-pool problem and derives the decision to engage in criminal activities. The equilibrium conditions are laid out for given protection expenditures. In section 4, protection decisions are derived for each individual. The system of equilibrium equations for the entire crime market is presented in section 5, which accounts for the equilibrium between the aggregate supply and demand.
for crime and the optimal protection decisions of individuals. Some predictions of the model are derived in section 6 while a few policy implications are considered in section 7. The conclusion proposes some extensions.

2 Some facts and issues on property crime

Our intent here is not to conduct an extensive survey of the literature on crime, but rather to mention a few empirical observations, with some theoretical results, that will provide a framework of discussion for the ensuing analysis.\textsuperscript{6}

The obvious question to ask is what motivates individuals to commit crimes. The literature on this being huge if one considers all the social sciences, we restrict ourselves to economic incentives. The fact that criminals respond to economic incentives is beyond controversy today for most crimes and especially property crimes. Becker (1968) was certainly the first to analytically formalize the idea that the supply of offenses by individual criminals is the result of a comparison between the marginal gains and costs of committing an additional crime, where the costs increase with the probability of detection and severity of punishment.\textsuperscript{7} Ehrlich (1973) then attempted to confront this theory with observations and did find that crime rates were negatively related to the probability and severity of punishment. Further studies tend to confirm the significant, negative effect of probability of apprehension on crime; the effect of severity of punishment is, however, still the subject of some controversy (Eide 2000, 360).\textsuperscript{8,9}

Another important factor that affects the opportunity cost of engaging in crime is related to the legitimate wage opportunities of individuals. This cost includes lost wages while setting up and committing a crime, while in jail, and also the stigma attached to having received a sentence, which can


\textsuperscript{7}Polinsky and Shavell (2000) point out that Becker’s model constitutes a formalization of ideas expressed earlier by Jeremy Bentham in 1789.

\textsuperscript{8}Although more rare, some are still not convinced about the effect of probability of punishment. For instance, Anderson (2002) has found that, in the case of pick-pocketing, 76\% of active criminals do not perceive the risk of apprehension when committing a crime.

\textsuperscript{9}For a survey of issues of empirical methodology, see Ehrlich (1996) or Witte and Witt (2001).
tends to reduce future income opportunities. Freeman (1996, 1999) surveys the evidence that labor market opportunities do affect individual decisions to participate in crime. Moreover, Lott (1990) finds that criminals with higher preconviction incomes lose more in terms of post-conviction incomes, and that this is often true in both absolute and relative terms. He also estimates that this reduction in post-conviction income constitutes a major part of the economic penalty imposed on criminals. A short survey by Corman and Mocan (2002), concerning the most recent studies which attempt to correct for difficult methodological issues, confirms that both sanctions and economic conditions have a significant impact on crime, although which of the two has a larger effect remains an unresolved issue.

While few doubt that legitimate income opportunities affect individuals’ decisions to take part in crime, significant results seem to be harder to come by with aggregate data. This is probably partly because a general increase in income can have the two opposite effects of increasing the opportunity cost of crime in terms of lost wages, and increasing the returns to crime in terms of additional wealth to appropriate. The question is further complicated by the fact that public and private efforts at reducing crime are endogenous (Bourguignon, 1999). Soares (2002) and Bourguignon et al. (2002), for instance, find no specific link between crime rates and the level of per capita income. They do find a significant positive effect of inequality on crime however, even though previous evidence is also not convincing on this effect (Bourguignon et al., 2002).

The distribution of the burden of crime according to people’s wealth will also be considered in our model. Levitt (1999) addresses this issue using data from victimization surveys for the United States and finds that in 1974-75, poor households were less victimized by property crimes than richer households, but that this relationship was reversed in 1994-95. He attributes this change to increased investments in private protection by richer households as well as improved protection technology. Glaeser and Sacerdote (1999) also find that in the United States, households with higher income suffer lower chances of being victimized. Turning to Latin America, Gaviria and Pagés (2002) obtain a reversed relationship, i.e. higher income households are relatively more victimized than lower income ones. Kesteren et al. (2000) emphasizes the importance of the difference between macro- and micro-analysis, noting that poorer communities are usually associated with higher risk, but that within a community, richer individuals may be more at risk (p. 54).
The simple fact that a group is exposed to a lower crime rate does not necessarily imply that crime imposes a lower burden on this group. As Levitt (1999, p. 88) points out, the burden of crime should also include the costs of individual protection. Many believe that the private protection effect is quite important. Dilulio (1996), for instance, believes that differences in private protection investments explain an important part of the high victimization rates of people living in the inner-city areas of the United-States (p. 11). Unfortunately and for obvious reasons, there is very little evidence on this. Another similar problem when comparing the burden of crime on different income groups is that the empirical work on crime typically uses either victimization rates or reported crime rates as the measure of crime and thus does not account directly for the value of stolen property. For any given crime, this value can be different between income group and should be included in the equation of the burden of crime. Most probably because of lack of available data, we have not found any study that accounts for these differences.

A final issue to consider concerns the use of sanctions versus wealth redistribution as means of controlling crime. As mentioned above, reduced inequality does seem to reduce crime. But it does not mean that, from the point of view of wealthier individuals, redistribution is a more attractive way to reduce crime than heavier enforcement. Freeman’s (1996) survey suggests that most crime prevention programs manage to reduce crime, though the effect is usually modest. Moreover, targeting both high-risk youth and recently released prisoners just before they enter the job market can have a large effect relative to their costs, while early social interventions appear to be costly relative to their impact in the long term. But again, this does not tell us if the money would be better spent on the police force. Imrohoroglu et al. (2000) address this question in their general equilibrium model calibrated to fit United States data. They find that at the pre-existing equilibrium, redistribution is ineffective since it actually increases the crime rate because of its highly distortionary effect on the labor market.

3 Property crime as a “common-pool” problem

Each individual in the population is indexed by his wealth (or income) level \( a \). The population is distributed according to \( G(a) \), with support \((\underline{a}, \overline{a})\). Wealth
level $a$ is perfectly observable by all.\footnote{As we will argue in remark 2, we do not need to explicitly distinguish between wealth and income levels. In the following, the term wealth is used to simplify the exposition, but also refers to income.}

3.1 The supply of criminal activities

Each individual can supply up to one unit of his time to criminal activities. The opportunity cost of doing so is equal to a fraction of his wealth $\lambda a$. This assumption can be justified as the lost wages from legitimate employment or as the expected cost of being caught and punished (see remark 1). In a large economy, we make the assumption that as far as one individual is concerned, the return from each unit of time spent on illegal activities is constant. The opportunity cost being also constant, the choice is really all or nothing: if an individual finds that crime pays, he will spend his whole unit of available time on it. Denote the return from each unit of criminal activities as $v$. Then, for an individual of type $a$, crime will pay as long as

\begin{equation}
(1) \quad v > \lambda a.
\end{equation}

\textbf{Remark 1} Instead of being a fraction of his wealth, the opportunity cost of criminal activities could be interpreted as the probability of being caught and punished. Let $p$ be that probability and assume that punishment is equal to a fraction $1 - \eta$ of an individual’s ex-post wealth, with $\eta \in (0, 1)$. In this case, crime pays if

\begin{equation}
(2) \quad (1 - p)(a + v) + p\eta(a + v) > a \quad \text{or, equivalently,} \quad v > \frac{p(1 - \eta)}{1 - p(1 - \eta)} a.
\end{equation}

For fixed $p$, the decision to become a thief is thus analogous to the previous one in (1).

\textbf{Remark 2} If punishment takes the form of imprisonment, it can be argued that the wealthier have a higher opportunity cost of jail time, in monetary equivalent, even for those who do not work. Indeed, the rich could always use its wealth to buy land, or a business, so as to attain a higher labor productivity. The high-income person would also lose more in terms of lower, post conviction income opportunities due to reputation effects (see, for instance, Lott, 1990). And even if the wealthier person could “buy” justice by affording better defense (Lott, 1987), paying a bribe to an enforcement officer or
influencing a judge, the outlay is likely to be larger the wealthier the person is.

Due to personal characteristics, some individuals will not engage in criminal activities, regardless of their wealth status. We denote the proportion of potential criminals as \( \alpha, \alpha \in (0, 1) \). The total amount of criminal activities will be defined by the marginally indifferent criminal \( \tilde{a} \) for which

\[
(3) \quad v = \lambda\tilde{a}.
\]

As a result, the total supply of criminal activities will be, for any given \( v \),

\[
(4) \quad x_s = \alpha G(\tilde{a}) = \alpha G\left( \frac{v}{\lambda} \right).
\]

3.2 The “demand” for criminal activities

The amount of an individual’s wealth lost to crime is a function of both his protection expenditures, \( y_a \), and the total amount of time, \( x_a \), that criminals spend trying to appropriate from him. This leads us to introduce an appropriation technology as follows:

**The appropriation function**  The total expected share of individual \( a \)’s wealth appropriated by criminals will be represented by appropriation function \( \gamma(x_a, y_a) \in (0, 1) \), where \( x_a \) and \( y_a \) are, respectively, the total amount of time that criminals spend trying to appropriate from individual \( a \), and the total protection effort that he expends. \( \gamma(x_a, y_a) \) is assumed homogeneous of degree zero with \( \gamma_x > 0, \gamma_{xx} < 0, \gamma_y < 0, \gamma_{yy} > 0 \) and \( \gamma(0, y_a) = 0 \) for all \( y_a \).

The zero homogeneity assumption implies that the appropriation function depends only on the ratio of efforts \( y_a / x_a \). Hence, if the levels of crime and protection efforts are both increased by the same factor, the share of expected wealth lost remains the same. \(^{11}\)

Note that we interpret \( x_a \) as the total number of hours spent trying to take from \( a \). This includes information gathering about a victim’s habits, his

\(^{11}\)This type of appropriation function is referred to as a contest success function in the literature on conflicts. There are two large classes of such functions: the ratio form, adopted here, and the difference form, which holds that the degree of success depends on the difference of efforts rather than the ratio. In the present application, the ratio forms is certainly much more appropriate. On a discussion about contest success functions, see Skaperdas (1996) and Hirshleifer (1989).
protection level, the location of his belongings, etc, as much as the eventual break-in, or attack, time. This means that even if tagged with a positive \( x_a \), a target might never actually lose anything. The fact remains, though, that the higher the \( x_a \), the larger the expected loss. A symmetrical argument is made about a criminal’s time.

For an individual of wealth \( a \), the expected unit return from crime at his place is thus \( a \gamma(x_a, y_a)/x_a \). We adopt the common-pool assumption that the average return from crime at any location be the same. This is a safe assumption to make as one would not expect all the criminals in a society to get together in order to increase the efficiency of their criminal activities (except, maybe, in the case of organized crime, which we do not consider here).\(^{12}\) This implies, for instance, that some criminals may spend more effort in searching and preparing to take from a wealthier person (a larger fish), and/or organize the take in a group which will split the proceeds, while others will act in solo and/or take mostly at random.\(^{13}\) The upshot is that one would expect that in equilibrium, from the point of view of criminals, the average return per victim is equalized.\(^{14}\)

Having expressed that expected return as \( v \), we must have, for any \( x_a > 0 \),

\[
(5) \quad \frac{\gamma(x_a, y_a)}{x_a} a = v.
\]

This equality defines an implicit relation between \( x_a \) and \( v \) for an individual of wealth \( a \) who spends \( y_a \) protecting his wealth. Let us express this implicit

\(^{12}\)The original work on the common-pool problem is that of Gordon (1954). The basic idea is that if a fisherman can freely choose between many fishing grounds, he will go to the one with the highest average productivity. In equilibrium, fishermen will thus allocate their efforts so that all fishing grounds offer the same average productivity. This is considered inefficient as efficiency calls for an equalization of marginal productivities. Gordon’s model, of course, assumes that no restrictions are placed on accesses to the grounds, which is often not the case as entry can be regulated by the state or by a common agreement between the fishermen. In our model of crime, each house can be assimilated to a different “fishing ground” in Gordon’s analysis.

\(^{13}\)Glaeser and Sacerdote (1999) hold a similar line of argument in that “… the returns per crime will rise with density as criminals choose only the more promising victims or criminals will select more victims and the returns per hour of criminal activity will rise with density.” (p. S241)

\(^{14}\)In the case of Shavell’s (1991) analysis, this assumption would correspond to the case of observable protection.
relation as \( x(a, v, y_a) \). The total “demand” for criminal activities is thus

\[
(6) \quad x_d = \int_a^\lambda x(a, v, y_a)g(a)\,da.
\]

3.3 The equilibrium supply and demand for crime

In equilibrium, the number of individuals being pushed into crime must be equal to the total number being pulled into it, \( x_s = x_d \), or\(^{15}\)

\[
(7) \quad \lambda G\left( \frac{v}{\lambda} \right) = \int_a^\lambda x(a, v, y_a)g(a)\,da.
\]

This equilibrium is represented in figure 1 for given \( y_a, a \in (\underline{a}, \bar{a}) \). It is straightforward to check that the equilibrium exists and is unique. For low enough \( \lambda \) or \( y_a, a \in (\underline{a}, \bar{a}) \), one can get equilibria in which all the potential criminals engage in crime. Conversely, one can find a case where either \( \lambda \) or \( y_a, a \in (\underline{a}, \bar{a}) \), are large enough to eliminate crime. Note that this last possibility is ruled out if \( \lim_{x_a \to 0} \gamma(x_a, y_a)/x_a = +\infty \).

4 The protection problem

We assume that individuals are neutral towards risk and thus seek to maximize the value of their wealth, net of protection expenditures and the expected share lost to criminals. Each individual is unable to affect \( v \), as it is set by relation (7) in the larger economy. However, given \( v \), an individual indirectly sets the level of predation against him through the choice of his protection level as described by equation (5). His program can be summarized by the following:

\[
(8) \quad \max_{y_a} V_a = a - \gamma(x_a, v, y_a, y_a)a - y_a,
\]

The first-order condition for this problem is:

\[
(9) \quad \frac{\partial V_a}{\partial y_a} = -a \left( \gamma_x \frac{\partial x_a}{\partial y_a} + \gamma_y \right) - 1 = 0,
\]

\(^{15}\)This equilibrium concept of crime fits in well with the following remark by Dr. Witte’s son made about drug dealers in New York: “Mom, they are like cockroaches – as soon as one leaves there is another to replace him.” (Witte and Witt, 2001, p. 18)
where

\[
\frac{\partial x_a}{\partial y_a} = \gamma_y \frac{x_a}{x_a - \gamma_x},
\]

as per the implicit relation between \(x_a\) and \(y_a\) given in (5). The solution to the first-order condition determines the final wealth level \(V_a^*\) of an individual of initial wealth \(a\), that is, net of expected stolen wealth and protection expenditures. In the following, we shall refer to \(V_a^*\) as the useful wealth.

5 A market-for-offenses equilibrium

Equations (5), (7) and (9) are the three equations that fully describe the market-for-offenses equilibrium. The first determines how much predation effort a victim will be subject to, given its protection effort \(y_a\) and the overall return to crime; the second matches the total supply and demand for crime in the economy; and the third represents the condition describing each potential victim's optimal protection effort. The endogenous variables are \(x_a\) and \(y_a\), \(a \in \{a, \bar{a}\}\) and \(v\).

It turns out that the analysis of the equilibrium system can be greatly simplified using the homogeneity properties of the appropriation function. By the homogeneity of degree zero, we have \(\gamma(x, y) = \gamma(1, y/x)\). Let us introduce function \(\rho(r) \equiv \gamma(x, y)\), where \(r\) is the protection-predation effort ratio \(y/x\). The common-pool equilibrium condition (5) implies that

\[
x_a = \frac{\rho(r_a)}{v} a.
\]

Since \(\gamma_y \equiv \rho'(r_a)/x_a\), the victim’s first-order condition becomes

\[
\frac{\rho(r^*)}{\rho'(r^*)} + r^* = -v,
\]

Since \(v\) is set in the general economy, equation (12) implies that in equilibrium, \(r_a\) will be the same for all potential victims in the economy, i.e. the protection-effort ratios will be the same for all. We can therefore drop
subscript \( a \) in the ensuing analysis. The system simplifies to

\[
\begin{align*}
(13) & \quad \alpha G \left( \frac{v_c}{\lambda} \right) v_c = \rho (r^*) \dot{a}, \\
(14) & \quad \frac{\rho (r^*)}{\rho'(r^*)} + r^* = -v_c, \\
(15) & \quad x^*_a = \frac{\rho (r^*)}{v} a, \\
(16) & \quad y^*_a = r^* x^*_a,
\end{align*}
\]

where \( \dot{a} = \int_a^a \alpha g(a) da \) is a parameter that represents the total wealth in the economy (or mean wealth level since the total population has been normalized to one) and subscript \( e \) refers to an equilibrium value. Expressed this way, the left-hand side of equation (13) represents the aggregate value of stolen property received by thieves and is strictly increasing in \( v \). It is represented in figure 2 by curve \( X_S \equiv \alpha G \left( \frac{v}{\lambda} \right) v \). The right-hand side of (13) represents the aggregate value of stolen property lost by the victims and, making use of the fact that \( r^* \) depends only on \( v \) as per equation (14), it is strictly decreasing in \( v \). It is represented in figure 2 by curve \( X_L \equiv \rho (r^*(v)) \dot{a} \).

Hence, the equilibrium exist and is unique. It is shown in figure (2), where the equilibrium values \( X_e \) and \( v_e \) are, respectively, the aggregate value of stolen property and the gross returns to crime.

6 Predictions of the model

One should note, first, that from equations (15) and (16), we get \( y^*_a = r^* \rho (r^*) a / v_e \). Combine this with the fact that all individuals in the economy will protect themselves so as to be subject to the same ratio of protection-to-predation effort \( (r^*) \), and we get the following expression for the equilibrium useful wealth of an individual of initial wealth \( a \):

\[
(17) \quad V^*_a = \left[ 1 - \rho (r^*) - \frac{r^* \rho (r^*)}{v_e} \right] a.
\]

By the envelop condition, this means that, for any parameter value \( z \) of the model, the total derivative of useful wealth will be

\[
(18) \quad \frac{\partial V^*_a}{\partial z} = \left[ 1 - \rho (r^*) - \frac{r^* \rho (r^*)}{v_e} \right] \frac{\partial a}{\partial z} + \frac{r^* \rho (r^*)}{v_e^2} \frac{\partial v_e}{\partial z} a \frac{\partial v_e}{\partial z}.
\]
Individuals will be ultimately concerned about how any policy, or event, affects only two variables: (i) their own initial wealth, \( a \), and (ii) the equilibrium gross returns to crime, \( v_e \). We will refer to the first effect as the *initial-wealth effect* and the second as the *deterrence effect*. The sign of the first effect is obvious. But the reason why any increase in the returns to crime \( v \) has a positive effect on a victim’s net wealth is because it makes it relatively less attractive. This can be most clearly seen from equation (5) where, for given \( y_a \), any increase in \( v \) will reduce the proportion of his wealth lost to crime. Intuitively, the increased returns to crime in the rest of the economy makes one’s own wealth relatively less attractive in equilibrium, thereby making it easier to protect. Note that this result contrasts slightly with the common approach in both the empirical and theoretical literature which usually uses crime rates as the relevant variable assumed to affect people’s welfare.\(^{16}\) As a general rule, any policy or event that increases \( v_e \) will be beneficial to anyone, as long as it does not affect the initial wealth too severely.

### 6.1 Comparing the rich and the poor in the economy

From equation (17), the equilibrium ratio of useful-to-initial wealth \( V^*_o / a \) will be the same for all individuals, and so will the ratios of predation effort, \( x^*_o / a \), and protection effort, \( y^*_o / a \). This means that regardless of initial wealth, all individuals lose the same share of their wealth to crime, when losses include both stolen property and protection efforts. Now if the equilibrium appropriated share, \( \rho (r^* ) a \), were seen as a sure loss and poor people suffered more heavily from loosing, say, 5\% of their wealth than rich people (the equivalent of decreasing relative risk aversion), then one might conclude that in equilibrium, wealthier individuals are less severely affected by crime than poorer ones. Would this explain the higher sensitivity of poorer classes to security issues in the 2002 French elections? Note also that absolute predation and protection efforts, \( x^*_a \) and \( y^*_a \), are both increasing in initial wealth, \( a \), and at the same rate.

\(^{16}\)It also fits in well with one of the authors’ argument that since his car is quite average (and dented and rusty), within the Belgian car pool, he is not worried about getting it stolen. He would be a little more worried if he were using the same car in Latin America though.
6.2 Economic growth

Economic growth causes an increase in total wealth $\dot{a}$ and a variation of wealth distribution function, which will now be expressed as $G(a; \dot{a})$. Graphically, it causes an upward shift of curve $X_L$, and a movement of curve $X_S$ which will depend on how the fruits of economic growth are distributed among the population. Figure 3 presents one possibility in which even though the effects of economic growth are fairly well distributed among the population (curve $X_S$ shift to the right to $X'_S$ by roughly the same amount everywhere), the total value of stolen property increases.

As can be seen in equation (28) of Appendix A, a sufficient condition for the returns to crime ($v$) to increase is for $G_a \left( \frac{v}{\lambda}; \dot{a} \right)$ to be non-negative, i.e. growth does not reduce the wealth of the marginal thief ($\dot{a}$), which is a reasonable assumption to make. This being the case, one can see from equation (18), where variable $\varepsilon$ becomes $\dot{a}$, that all those who do not directly benefit from economic growth will still get an indirect kickback through the effect on crime. So even though $\partial a / \partial \dot{a}$ is nil for some individuals, the increased wealth in the rest of the economy makes one’s own wealth relatively less attractive in equilibrium, thereby making it easier to protect. An unequal economic growth in which does not benefit the poorer segment of the population could still be beneficial for them as their useful wealth increases.

6.3 Income distribution and crime

We consider first the effects of a change in income distribution for given total wealth ($\dot{a}$). Since total wealth does not change, the aggregate amount that victims lose to crime, given $v$, is not affected by a change in the distribution of income. Curve $X_L$ thus remains unchanged (see figure 2). The total amount stolen by thieves, given $v$, will however be affected since a change in the distribution function $G(a)$ affects curve $X_S$. It is clear, from figure 2, that whether the equilibrium level of crime $X_e$ increases or not will depend solely on how the initial income of the indifferent criminal, $\ddot{a} = v_e / \lambda$, is affected by the change. If he gets richer, then $G(\ddot{a})$ moves down and his opportunity cost of participating in crime becomes larger. This makes him an honest person. $X_S$ moves down at $v_e$ and the equilibrium crime level $X_e$ is reduced. The opposite will of course take place if the indifferent criminal becomes poorer.

For concreteness, let us assume that inequality is reduced in the sense that income distribution $G(a)$ becomes more concentrated around its mean.
ˆa. It is shown in figure 4 that crime will decrease if the indifferent criminal has lower initial wealth than the average wealth, i.e. ˆa < ˆa (compare the initial thin curve with the thick curve). Otherwise, crime will increase, as depicted by the dotted line which corresponds to a case of reduced inequality when ˆa > ˆa. This result accords well with Bourguignon et al. (2002), who emphasize the importance of concentrating on a specific part of the income distribution curve in order to verify if inequality does affect crime. They find that in the case of Colombia, “that part of the population which most matters for time fluctuations in the crime rate are thus those individuals whose welfare lies below 80 percent of the mean of the whole population.” (p. 8) According to our model, we leave it as an open question if this implies that the marginal criminal in Colombia has an income of 80% of the mean.

7 Policies to reduce crime

7.1 Increasing the opportunity cost of crime or the public enforcement effort

For any individual, the opportunity cost of crime was assumed equal to λa, where λ could be regarded as a measure of the public enforcement effort. It is shown in appendix B that the implicit relation between r and λ is positive. This simply means that more public enforcement results in less predation relative to the protection effort, with the result that less wealth will be stolen in equilibrium, i.e. ρ(r*) decreases. In fact, an increase in λ causes a rightward shift of curve XS between λa and λa (see figure 2), while X_L does not depend on λ. Hence, the gross returns to criminal activities (v_c) increase with the public enforcement effort, while the value of stolen property (X_e) goes down. The effect on the reduction of stolen property will be more important the larger is the slope of X_L, or the smaller is the slope of X_S. For instance, the larger the total wealth (a), the larger the effect of a variation of λ on X_c, *ceteris paribus*.

As for the effect on individuals, it is clear that since v_c increases with λ, they are all better off (see equation (18) and the comments preceding it). But such an increase in λ, if caused by an increase the public enforcement effort, would normally be achieved through an increase in taxation, and thus, a reduction in initial wealth (a). From the second term on the right-hand side of (18), we see that the richer the individual, the larger the gain from an increase in v_c. Consequently, richer individuals would be willing to pay more for such additional public protection since in the first term on the right-hand
side of equation (18), the part between brackets is the same for all (assuming a uniform tax rate).

7.2 Income redistribution

7.2.1 Redistributing income in order to contain crime

It is sometimes argued that a redistribution of wealth may be used as a means to reduce crime. The previous analysis implies that a necessary condition for this is that the indifferent criminal becomes richer after the redistribution. In that case, we have seen in figure 4 that the gross returns to crime $v_c$ would increase, which has a beneficial effect to all as per the deterrence effect in equation (18). Whether individuals are overall better off or not will depend on how their initial wealth is affected (the initial-wealth effect in (18)). For the poorest individuals, a redistribution scheme should make this last term positive, so that they gain on both counts. As for wealthier individuals who are net contributors to the redistribution scheme, they face a tradeoff between a reduction in their initial wealth and the reduced crime levels. Whether they gain or not, and whether they prefer redistribution to direct public enforcement in order to reduce crime, will depend on a host of parameters. In order to keep the exposition simple, let us concentrate on a specific redistribution scheme.

7.2.2 Uniform tax and lump-sum redistribution

Let us assume that individual wealth is taxed at rate $\tau$ and that the proceeds are redistributed equally between all as a lump-sum transfer. This leads to the following “mean-preserving contraction” of individual wealth $a$:

$$a' = (1 - \tau) a + \tau \bar{a}. \quad (19)$$

Obviously, in a society where crime is so endemic that the marginal criminal is richer than average wealth, such a redistribution scheme will not be an option to reduce crime, as it will make the marginal criminal poorer. It is shown in Appendix C that as tax rate $\tau$ increases, $v_c$ decreases if, and only if, the marginal thief is poorer than average (see equation (41)).

The equilibrium welfare of an individual initially endowed with wealth $a$ is now

$$V_a^* = \left[1 - \rho(r^*) - \frac{r^* \rho(r^*)}{v_c}\right] (a(1 - \tau) + \tau \bar{a}) . \quad (20)$$
For an individual, the marginal effect of increasing tax $\tau$ is thus

\begin{equation}
\frac{dV^*_a}{d\tau} = \left[ 1 - \rho(r^*) - \frac{r^* \rho(r^*)}{v_e} \right] (\bar{a} - a) + \frac{r^* \rho^*}{v_e^2} \frac{dv_e}{d\tau}.
\end{equation}

Again, it all revolves around a comparison between the initial-wealth and the deterrence effects. In Appendix C, we show that, ceteris paribus, the deterrence effect is larger, and therefore the share of the population that gains from the redistribution is larger, the larger the density around the marginal criminal, $g(v_e/\lambda)$ (see equation (42)), and the smaller the wealth of the marginal criminal compared to average wealth, $\bar{a} - v_e/\lambda$ (see equation (41)). It is interesting to see that Bourguignon et al. (2002) had noted the importance of the population density that lies below a certain income range in order to explain the effect of inequality on crime. The subtle difference with our results is that what seems to matter most for changes in crime rates is the density around the marginal criminal, not below.

### 7.3 Fighting crime with redistribution versus public enforcement

A perennial subject of discussion in the literature on crime is whether the state’s money would be better used in a redistributive scheme, rather than direct police enforcement, as a means of reducing crime. In order to analyze this question, let us assume that the state has a fixed budget to spend to fight crime, which we normalize to one, and consider that a share $\theta$ of that budget is earmarked for redistribution, while the balance goes to direct public enforcement. Hence, an increase in $\theta$ simply boils down to taking money away from the police and redistributing it to some individuals. We therefore introduce a public enforcement function with decreasing returns: $\lambda = \lambda(\theta)$, $\lambda'(\theta) < 0$ and $\lambda''(\theta) < 0$. Now one could think of many ways to redistribute wealth, some more efficient than others to contain crime. We choose to consider two polar schemes: the first one, referred to as non-targeted, redistributes evenly and non-discriminatively among a certain range of the poorest segment of the population; the second, which we will call targeted, specifically targets the marginal criminals. The second scheme is of course the most effective way of redistributing wealth in order to reduce crime. We choose to consider both schemes for realism’s sake, knowing that in practice, the marginal criminal will only imperfectly be targeted. Considering the less efficient and the most efficient schemes will allow us to draw conclusions...
about any intermediate situation.\footnote{17}{Note that the type of redistribution that we have in mind could is more than just collecting taxes revenue and redistributing it to the different agents. It should also be interpreted as investments in human capital through subsidized schooling or training programs for the less qualified, universal access to public health services, playgrounds in urban areas, positive discrimination, etc.}

7.3.1 Non-targeted redistribution versus public enforcement

The total crime fighting budget and the total population being normalized to one, we posit that the share $\theta$ of the crime fighting budget is redistributed evenly among proportion $\nu$ of the poorest individuals, which strictly includes the marginal criminal. The initial wealth of any receiving individual is thus expressed as $d^' = a + \theta/\nu$. As for wealthier individual, since the crime fighting budget is fixed, their initial wealth remains unchanged so that only the second term on the right-hand side of equation (18) will be affected by a change in $\theta$, i.e.

$$\frac{\partial V^*_a}{\partial \theta} = \frac{r^* \rho(r^*)}{v^*_c} \frac{\partial v_c}{\partial \theta},$$

where $\partial v_c/\partial \theta$ is derived in equation (44) of Appendix D. We obtain that increasing direct public enforcement to the detriment of redistribution is desirable as long as $\partial v_c/\partial \theta > 0$ which is the case if, and only if,

$$-\frac{v_c}{\lambda^2} \lambda(\theta) - \frac{1}{\nu} < 0.$$  

The intuition is simple; at the margin, a unit increase in $\theta$ will reduce crime if, and only if, the increase in wealth of the marginal criminal, $1/\nu$, exceeds the increase in $v_c/\lambda(\theta)$, given $v_c$. If this is the case, the formerly indifferent criminal will indeed become \textit{strictly} honest.

Now since $\nu$ is considered fixed, this suggests that an economy characterized by a lower $v_c$ is more likely, given $\lambda(\theta)$, to resort to additional redistributive policies instead of direct public enforcement. We have seen in section 6.2 on economic growth that an economy with a lower aggregate wealth will end up with a lower $v_c$, \textit{ceteris paribus}. This suggests that in poorer economies, a relatively more intensive use of redistribution policies of the non-targeted type compared to public enforcement may be more effective. We have seen also in section 7.2 that $v_c$ tends to be lower in an economy
where income is more equally distributed and crime is endemic in the sense
that the marginal criminal is richer than the average person, \( \bar{a} > \hat{a} \) (see fig-
ure 4). In such economies, therefore, redistribution of the non-targeted type
may also be preferred to public enforcement. The converse holds true for an
economy where income is more equally distributed crime and is non-endemic.

Finally, we have seen in Appendix B that \( v_c \) increases with \( \lambda \). The model
therefore does not have enough structure to predict whether ever increasing
spending on the police, to the detriment of a non-targeted redistributive
policy, will eventually be counterproductive.\(^{18}\) Corner solution, where either
none or the entire crime-fighting budget is spent on the police, cannot be
ruled out with a non-targeted redistributive policy.

7.3.2 Targeted redistribution versus public enforcement

For the sake of simplicity, let us assume that indifferent criminals are hon-
est. Obviously, the most efficient way to redistribute income in order to
reduce crime is achieved by precisely targeting those criminals whose wealth
is just below the marginal criminals, since any small amount will make them
honest. The trick is thus to give the lowest amount to a criminal which is
sufficient to turn him into a marginal criminal. It is shown in Appendix
D.2 that as \( \theta \) increases, i.e. more money is taken from the police force and
redistributed to the marginal criminals, \( v_c/\lambda \) must increase. Now since \( v_c/\lambda \)
represents the wealth of the marginal criminal, this means that as \( \theta \) increases,
the wealthiest criminal, still denoted \( \hat{a} \), is now strictly poorer than the in-
different criminal. An efficient policy of targeted redistribution is thus to
give money to the wealthiest criminal in order to turn him into an indifferent
criminal, i.e. he receives an amount \( v_c/\lambda - \hat{a} \). As a result, this policy cre-
at a positive mass of individuals whose wealth equals \( v_c/\lambda \). The resulting
wealth distribution is shown in figure 5. The cost of such a redistribution
scheme is \( \int_{\hat{a}}^{\infty} \left( \frac{v_c}{\lambda} - a \right) g(a) da \). Since the total budget earmarked for crime
fighting redistribution is \( \theta \), the general crime equilibrium must now respect

\(^{18}\) Introducing more structure to our model by using more specific functional forms goes
beyond our original intent for the present study. It is the object of ongoing research of
ours.
the following two equations

\begin{align}
(24) \quad \theta &= \int_{\hat{a}}^{\nu_e} \frac{\nu_e}{\lambda} \left( \frac{\nu_e}{\lambda} - a \right) g(a) \, da, \\
(25) \quad \alpha G(\hat{a}) \nu_e &= \rho(\nu^*(\nu_e)) \hat{a},
\end{align}

where \( r^*(\nu_e) \) is still given by first-order condition (14). Note that without
redistribution, \( \theta = 0 \), thereby making equation (24) irrelevant as \( \hat{a} = \nu_e/\lambda \),
and equation (25) becomes equivalent to the original equilibrium condition (13).

It is shown in Appendix D.2 that \( \nu_e \) increases with \( \theta \) if, and only if,

\begin{equation}
(26) \quad \frac{\nu_e}{\lambda^2} \lambda'(\theta) < \frac{1}{\int_{\hat{a}}^{\nu_e} g(a) \, da}.
\end{equation}

This condition is remarkably similar to condition (23) obtained in the case
of non-targeted redistribution: at the margin, a one unit increase in \( \theta \) will be
effective in fighting crime if, and only if, it raises the wealth of the indifferent
criminals at a pace faster than that of \( \nu_e/\lambda(\theta) \), for given \( \nu_e \). There is one
difference with non-targeted redistribution though, it is that with targeted
redistribution, the optimal value of theta is comprised strictly between zero
and one (see proof in Appendix D.2). As a result, with perfectly well targeted
redistribution, a crime fighting budget should always include a mix of income
redistribution and public enforcement.

8 Discussion and extensions

Our aim with this study was to fill a gap in the existing literature by intro-
ducing private incentives to invest in protection in a market-for-offenses
model of crime. In order to achieve this, it was necessary to make use of an
appropriation function of the type commonly used in the economic analysis
of conflict literature. We also had to make assumptions about the returns
to crime at different locations since the crime victims were heterogeneous in
their wealth levels and protection decisions. We made the simplifying as-
sumption that in equilibrium, the average (expected) returns to crime had
to be the same for all victims.

Our main results indicate that what matters for potential victims is the
gross returns to crime in the global economy. Moreover, the poor and the
rich lose the same share of their wealth to crime, and similarly for the share spent on protection. Which of the two groups will be worst affected by crime in equilibrium will depend on their marginal utility schedule. Both economic growth and inequality have ambiguous effects on crime. The model is however helpful in underlining which are the important parameters in this respect. We nonetheless show that no matter how the fruits of growth are distributed across the community, all individuals will benefit from growth because of the ensuing reduction in the relative severity of the crime problem. We also determine which are the important parameters to consider when implementing a redistribution policy aimed at reducing crime, and show that when redistribution can be well targeted, it will be optimal to use a combination of redistribution and public enforcement in order to reduce crime.

Although the analysis has provided us with some useful and intuitively appealing results, we believe that our model lends itself readily to various extensions to study other crime-related issues. For instance, we did not explicitly account for the fact that individuals can normally choose between crime and work, the latter having a wealth creating effect. Such an analysis could help us clarify the two-way relationship that may exist between crime and growth. There is also the question of simultaneous participation in both the legitimate labor and crime markets, which is sometimes observed, and may be introduced using decreasing returns to crime at the individual level. One may want to use our set-up to analyze the degree of complementarity and substitutability between private and public protection. And finally, our public enforcement function, when interpreted as the probability of catching a criminal, did not account for the number of criminals. One would think that for a given size of the police force, increasing the number of criminals should reduce the probability of catching each criminal. This could be achieved by introducing a police-to-criminal matching function of the type used in Furlong (1987) and, in a similar fashion to him, the unit cost of the police force should somehow be related to the average wealth of the economy.
APPENDIX

A Economic Growth

Let

(27) \[ \psi^A \equiv \alpha G \left( \frac{v_e}{\lambda}; \hat{a} \right) v_e - \rho(r^*(v_e)) \hat{a} = 0, \]

where \( r^*(v_e) \) is defined implicitly as per the first-order condition (14). Then

(28) \[ \frac{\partial v_e}{\partial \hat{a}} = -\psi^B \psi^B > 0 \]

(29) since \( \psi^B = \alpha G \left( \frac{v_e}{\lambda}; \hat{a} \right) v_e - \rho(r^*(v_e)) < 0 \) if \( G_k \leq 0, \)

(30) \[ \psi^B = \alpha G \left( \frac{v_e}{\lambda} \right) v_e + \alpha G \left( \frac{v_e}{\lambda} \right) - \rho'(r^*) \frac{\partial r^*}{\partial v_e} \hat{a} > 0, \]

(31) and \( \frac{\partial r^*}{\partial v_e} = \frac{-1}{2 - \frac{\rho'(r^*)}{\rho(r^*)}} > 0 \) by SOC.

Hence,

(32) \[ \frac{\partial r}{\partial \hat{a}} = \frac{\partial r}{\partial v} \frac{\partial v}{\partial \hat{a}} > 0, \]

(33) \[ \frac{dX_e}{\partial \hat{a}} = \rho'(r^*) \hat{a} \frac{\partial r^*}{\partial \hat{a}} + \rho(r^*), \]

(34) \[ \frac{d}{\partial \hat{a}} \left( \frac{X_e}{\hat{a}} \right) = \rho'(r^*) \frac{\partial r^*}{\partial \hat{a}} < 0, \]

(35) \[ \frac{\partial \hat{a}}{\partial \hat{a}} = \frac{1}{\lambda} \frac{\partial v_e}{\partial \hat{a}} > 0. \]

B Increasing the opportunity cost of crime

Introduce (14) into (13) to get

(36) \[ \psi^A \equiv \alpha G \left( \frac{v}{\lambda} \right) v - \rho(r^*(v)) \hat{a} = 0. \]

Hence

(37) \[ \frac{\partial v_e}{\partial \lambda} = -\frac{\psi^A}{\psi^A} = \frac{\alpha g \left( \frac{v_e}{\lambda} \right) \left( \frac{v_e}{\lambda} \right)^2}{\alpha [g \left( \frac{v_e}{\lambda} \right) \frac{v_e}{\lambda} + G \left( \frac{v_e}{\lambda} \right)] - \rho'(r^*) \frac{\partial r^*}{\partial v_e} \hat{a}} > 0, \]

(38) \[ \frac{\partial r^*}{\partial \lambda} = \frac{\partial r^*}{\partial v_e} \frac{\partial v_e}{\partial \lambda} > 0. \]
Using the envelop condition, we get
\[ \frac{\partial V^*_a}{\partial \lambda} = \frac{\partial V^*_a}{\partial v} \frac{\partial v}{\partial \lambda} > 0. \]

C Uniform tax and lump-sum redistribution

The equilibrium with taxation is described by
\[ \psi^C \equiv \alpha G \left( \frac{v_e - \tau \hat{a}}{1 - \tau} \right) v_e - \rho(r^*(v_e))\hat{a} = 0. \]

Using the implicit function theorem, we have
\[ \frac{dv_e}{d\tau} = -\alpha \frac{\tau (\frac{v_e}{\lambda} - \hat{a})}{\frac{1}{\lambda(1 - \tau)} g(\cdot)} \frac{G(\cdot) - r^* \rho^r(r^*)}{r^* \rho^r(r^*)} > 0 \text{ iff } \frac{v_e}{\lambda} < \hat{a}. \]

We show in the following that the when the density around the marginal criminal \( g(\cdot) \) increases, the deterrence effect also increases, \textit{ceteris paribus}. The exercise is to compare the effect of additional redistribution for two economies characterized by the same \textit{pre-tax} equilibrium, while differing by the population density at the marginal criminal only. We have
\[ \frac{\partial (dv_e/d\tau)}{\partial g(\cdot)} = \tau \left( \frac{\hat{a} - v_e}{\lambda} \right) \frac{1}{\tau(1 - \tau)^2} \frac{G(\cdot) - r^* \rho^r(r^*)}{(\frac{1}{\lambda(1 - \tau)} g(\cdot) + G(\cdot) - r^* \rho^r(r^*))^2} > 0 \text{ iff } \hat{a} > v_e/\lambda. \]

Hence, the deterrence effect of an increase in redistribution is more important for an economy with a higher density at the marginal criminal.

D Redistribution or public enforcement?

D.1 Non-targeted redistribution

Since the initial wealth of a poor individual is now \( a' = a + \theta \nu \) and it includes the marginal criminal, we now have \( \hat{a} = v_e/\lambda - \theta /\nu \). The crime equilibrium condition becomes
\[ \psi^D \equiv \alpha G \left( \frac{v_e}{\lambda(\theta)} - \frac{\theta}{\nu} \right) v_e - \rho(r^*(v_e))\hat{a} = 0. \]

Hence,
\[ \frac{\partial v_e}{\partial \theta} = -\frac{\psi^D}{\psi^D} = \frac{\alpha g(\cdot) \left( \frac{v_e}{\lambda} \right) \lambda (\theta) + \frac{1}{\nu} v_e}{\alpha \left[ g(\cdot) \frac{v_e}{\lambda} + G(\cdot) \right] - \rho'(r^*) \frac{v_e}{\lambda} \hat{a}}. \]
D.2 Targeted redistribution

Proof that $v_e/\lambda$ is non-decreasing in $\theta$: We proceed by contradiction and suppose that $\frac{\partial}{\partial \theta} \frac{v_e}{\lambda} < 0$. Since $\lambda'(\theta) < 0$, this implies that $\frac{\partial v_e}{\partial \theta} < 0$ and thus $\frac{\partial}{\partial \theta} \left[ \alpha G \left( \frac{v_e}{\lambda} \right) v_e \right] < 0$. But since $\frac{\partial^*}{\partial V} \rho(r^*(v_e)) \hat{a} = \rho'(r^*) \frac{\partial^*}{\partial v_e} \frac{\partial v_e}{\partial \theta} \hat{a} > 0$. This violates general equilibrium condition (25). ♦

The implicit relation between $v_e$ and $\theta$: From (24) and (25), we introduce $F^1$ and $F^2$ such that

$$F^1 \equiv \int_{\hat{a}}^{\frac{v_e}{\lambda}} \left( \frac{v_e}{\lambda} - a \right) g(a) da - \theta = 0,$$
$$F^2 \equiv \alpha G(\hat{a}) v_e - \rho(r^*(v_e)) \hat{a} = 0,$$

where $R^*(v_e)$ is still defined by victims' first-order condition (14). Making use of the implicit function theorem, we have

$$(45) \frac{\partial v_e}{\partial \theta} = -\frac{F^1_{\theta} F^2_{\hat{a}} - F^2_{\theta} F^1_{\hat{a}}}{F^1_{v_e} F^2_{\hat{a}} - F^2_{v_e} F^1_{\hat{a}}},$$

where

$$F^1_{\theta} = -\frac{v_e}{\lambda^2} \lambda'(\theta) \int_{\hat{a}}^{\frac{v_e}{\lambda}} g(a) da - 1;$$
$$F^1_{v_e} = \frac{1}{\lambda} \int_{\hat{a}}^{\frac{v_e}{\lambda}} g(a) da \geq 0;$$
$$F^1_{\hat{a}} = -\left( \frac{v_e}{\lambda} - \hat{a} \right) g(\hat{a}) \leq 0;$$
$$F^2_{\theta} = 0;$$
$$F^2_{v_e} = \alpha G(\hat{a}) - \rho'(r^*) \hat{a} \frac{\partial r^*}{\partial v_e} > 0;$$
$$F^2_{\hat{a}} = \alpha g(\hat{a}) v_e > 0.$$

$\frac{\partial v_e}{\partial \theta}$ will thus be positive if, and only if, $F^1_{\theta}$ is negative. Moreover, as $\theta \to 0$, we have $\frac{v_e}{\lambda} \to \hat{a}$, with the result that $\frac{\partial v_e}{\partial \theta} \to +\infty$. And conversely, as $\theta \to 1$, we have $\lambda \to 0$, with the result that $F^1_{\theta}$ becomes strictly positive. Hence, $\frac{\partial v_e}{\partial \theta}$ is strictly positive for low values of $\theta$ while it is strictly negative for high values of $\theta$. 
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Figure 1: The Equilibrium supply and demand for crime
Figure 2: The Equilibrium Level of Crime
Figure 3: The Effects of Economic Growth on Crime
Figure 4: Income distribution and crime
Figure 5: Efficient targeted redistribution