0.1 The basic bio-economic model of the fishery

In this section, we introduce stock dynamics into the problem of the fishery. The dynamics are based on the biological mechanics of a fish population.[1]

0.1.1 The natural growth process

We begin by describing the natural growth of the fish stock in the absence of human activity. The proposed model is simplified in that it considers only one fish variety with no interaction between fish species, it does not account for the age of the fish, and it assumes that only the fish stock size affects the growth rate of the population. Our analysis is made in continuous time.[2]

First, let ρ denote the growth rate of the fish stock in the absence of any environmental constraint; that is, as if the availability of food were not a constraint to the population's growth. It is referred to as the *intrinsic* growth rate of the stock. With $S(t)$ as the fish stock at time t, we have

$$\dot{S}_t = \rho S_t, \tag{1}$$

where $\dot{S}_t \equiv \partial S_t / \partial t$ is the *rate of growth* of the fish stock while $\dot{S}_t / S_t$ is termed the *proportional* rate of growth . This implies that with an initial fish stock equal to S_0, we have

$$S_t = S_0 e^{\rho t}. \tag{2}$$

Clearly, the above is not a reasonable representation of the growth of a fish stock as it implies boundless growth. Due to a finite availability of nutrients as well as competition with other species, we wish to incorporate the fact that only a finite size of the fish stock can be carried by the environment. We call this the finite *carrying capacity* , denoted $\bar{S}$.

It moreover seems reasonable to posit that as the stock size increases, its proportional rate of growth decreases; that is, $\dot{S}_t / S_t$ is decreasing in S_t.

[1] The basic model which combines both population dynamics and economics is due to Schaefer 1957 and is often referred to in the literature as the *Schaefer model*.

[2] The logistic growth function for population growth that we will derive is attributed to Verhulst (1838).

Those properties are well represented by the following *logistic growth function* :

$$G(S(t)) = \rho\left(1 - \frac{S(t)}{\bar{S}}\right) S(t). \tag{3}$$

$G(t)$ represents the natural rate of change of the fish stock so that, in the absence of fishing activities, we have $\dot{S}(t) = G(t)$. The logistic form implies that the *proportional* natural growth rate ($\dot{S}/S$) is maximum when the population size is very small; this is due to the fact that environmental constraints are largely non-existent. Conversely, as the stock size approaches its maximum carrying capacity $\bar{S}$, growth falls to zero. One may note that $G(S)$ is quadratic; this implies that the absolute growth level has a unique maximum value, termed the *maximum sustainable yield* , at stock size $\hat{S}$ characterized by $G'(\hat{S}) = 0$. The implied evolution of the population size over time is illustrated in figure 1.

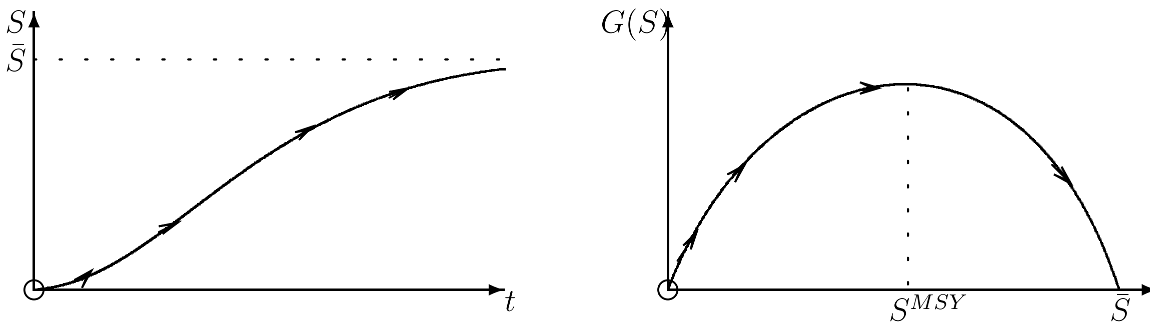

Figure 1: The logistic growth function

0.1.2 Fishing activities

In problem **??**, a simple case of fishing activities was introduced in terms of harvest level (done in class). From an economic standpoint, it would be more useful to instead consider effort levels as the decision unit of fishers, as it directly determines the costs of the activities. In practice, fishing efforts encompass a variety of input types such as the number of boat-hours spent at sea, the size and number of nets used, the size of a boat and the number of men operating on it, the radar equipment, etc. For now, we shall simply assume that effort level is the number of boats, denoted $X(t)$.

It would be unreasonable, however, to assume the harvest rate to be entirely determined by the effort level. Indeed, this would imply that at any effort level, one could catch the same amount of fish regardless of how many there are left in the water. For this reason, it is customary to assume that the harvest rate increases with the fish stock size. We thus posit the following harvest function:

$$y(t) = f(X(t))S(t) \text{ with } f'(X) > 0 \text{ and } f''(X) \leq 0. \tag{4}$$

This function implies that for a given effort level, the harvest rate increases linearly with the stock size. It is illustrated in figure 2, where lines $y(X_1, S)$ and $y(X_2, S)$ represent harvest levels with two different constant effort levels, X_1 and X_2 respectively, $X_1 < X_2$.

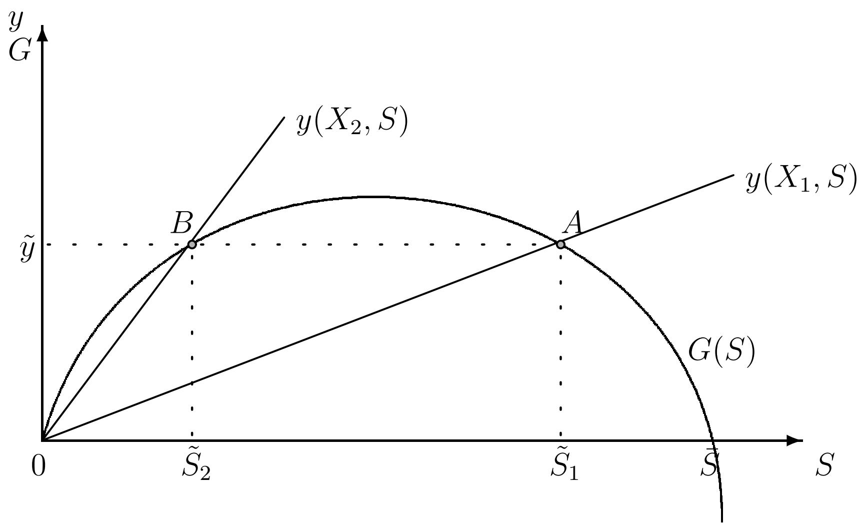

Figure 2: Logistic growth and harvesting with a constant effort

In order to apprehend the effect of constant fishing effort X_1, suppose that the stock size is initially at its natural long-run stationary state $\bar{S}$. An application of effort X_1 leads to a harvest rate that exceeds the natural growth of the resource since at stock size $\bar{S}$, we have $y(X_1, \bar{S}) > G(\bar{S}) = 0$. Hence, the stock of the resource will decline. Given effort rate X_1, one can see that the harvest rate exceeds the natural growth rate for all stock levels above $\tilde{S}_1$. Point A therefore represents a stationary state since the quantity harvested equals the natural growth rate, that is, $G(\tilde{S}_1) = y(X_1, \tilde{S})$. In other words, with effort level X_1, one obtains sustainable harvest rate $\tilde{y}$.

It should be noted that given a constant effort level X_1, point A corresponds to a *stable* stationary state. Indeed, if, for some reason, the stock size falls below $\tilde{S}_1$, its size begins to increase as the natural growth rate exceeds the harvest rate.

Analogous reasoning leads to point B as another stable stationary state corresponding to higher effort level X_2. Not surprisingly, a higher effort level leads to a steady-state stock size $\tilde{S}_2$ which is smaller than $\tilde{S}_1$. What may be less intuitive is that both effort levels lead to the same sustainable harvest level $\tilde{y}$. But the reason should now become clear: Even though effort level X_2 is higher than X_1, the associated lower steady-state stock size $\tilde{S}_2$ renders each unit of effort less productive than with a larger stock $\tilde{S}_1$. Each effect is exactly offset by the other to make the harvest rates equal.

The reader may have already noted that X_2 corresponds to an inefficient long-run effort level. Indeed, one can achieve the same harvest rate with a lower effort level X_1. This conclusion would be correct if one were to consider the purely static reasoning of comparing point A to point B, that is, as if changing the effort level would lead to an instantaneous jump from one steady-state to the other. But as we shall see below, when one considers a truly dynamic setting with the introduction of an interest rate, point B may become the socially efficient steady-state. The reason being that if one can invest the additional proceeds from the transition period in projects that yield a high social return, then it might be worthwhile to increase the effort to X_2.

0.1.3 The yield-effort curve

It should become clear from the above analysis that for any effort level X, one can find a corresponding steady-state harvest rate. This leads to the determination of the steady-state *yield-effort curve* as illustrated in figure 3.

The peculiar shape of the yield-effort curve is entirely due to the manner with which the natural rate of growth of the resource depends on its stock

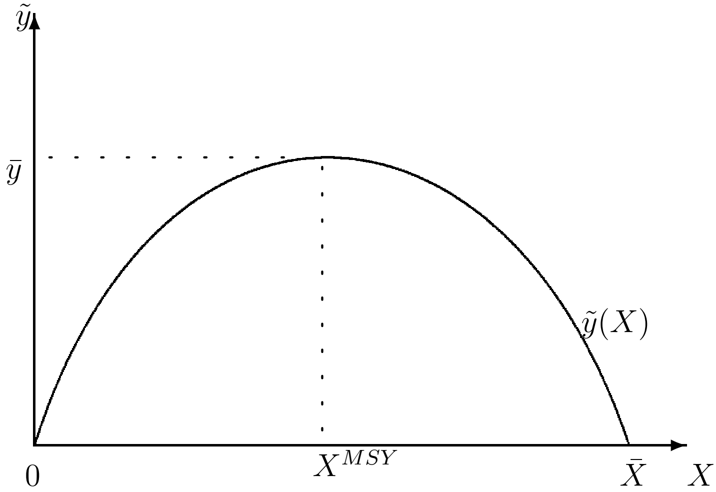

Figure 3: The steady-state yield-effort curve

size, which we shall refer to as a *stock effect* .[3] Indeed, even in the absence of congestion effects - i.e. $f''(X) = 0$ - the stock effect causes the yield-effort curve to be concave, initially increasing up to a *sustainable maximum yield* denoted $\bar{y}$, and thereafter decreasing to reach zero steady-state harvest at constant effort level $\bar{X}$. At low effort levels, an increase in the effort allows for a higher catch because a lower stock size corresponds to a higher natural growth, that is, $G'(S) < 0$ as can be seen in figure 2. The concavity of the yield-effort curve is due to the concavity of $G(S)$, which implies that natural growth increases are less important as S becomes ever smaller. As the stock size reduces to S^{MSY}, an increase in effort has no effect on output at the margin; this effort level corresponds to the maximum sustainable yield, as represented by coordinates X^{MSY} and y^{MSY}. For effort levels above X^{MSY}, lower stock sizes lead to lower natural growth since $G'(S) > 0$; increased effort produces lower steady-state output levels. Sustainable output eventually reaches zero at effort level $\bar{X}$, which corresponds to the minimum effort level for which the resource is driven to extinction.

Exercise 1 *Can you think of a situation where $\bar{X} \to \infty$?*

[3]See Smith (1968) for a discussion of the distinction between stock and congestion externalities.

0.1.4 Some bio-economic equilibria

In order to bring economics into the picture, suppose that each unit of the resource sells at constant price p and that each unit of effort has a constant unit cost c. Now a profit maximizing resource manager subject to a positive time discount rate would seek to maximize the present value of the resource. For now, however, some insight is gained by simply assuming that the objective of is to maximize the *per-period* profit rate. The steady-state total revenue function is given by $TR(X) = p\tilde{y}(X)$ while the total cost function is $TC(X) = cX$. Both are represented by curves TR and TC respectively in figure 4.

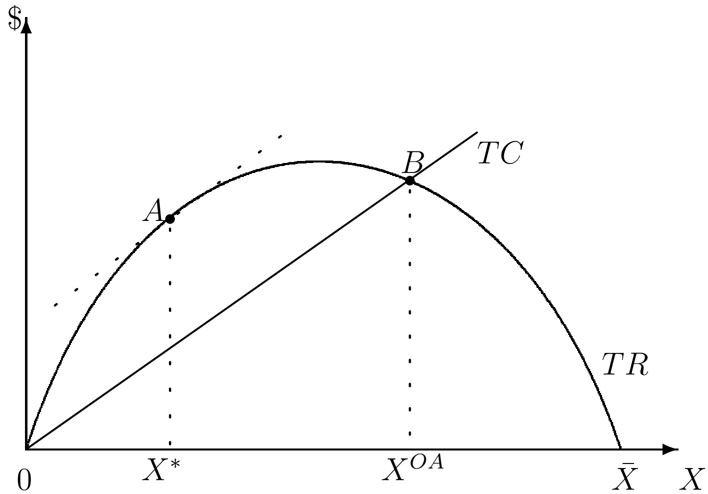

Figure 4: Bio-economic equilibria

Per-period profits are maximized at the effort level X^* which equates marginal revenues which marginal costs. We shall refer to this as the *restricted entry* equilibrium. The positive slope implies that $S^*(X^*) > S^{MSY}$. As a result, per-period profit maximization leads one to choose a sustainable harvest rate which falls short of the maximum sustainable yield.

The positive rents that one obtains at X^* imply that the average product of effort exceeds the unit cost c. As observed in section **??**, this suggests that in the absence of exclusion, others who can harvest at the same cost will be tempted to exploit the resource as well. An open-access situation leads to steady-state equilibrium exploitation level X^{OA} characterized by $p\tilde{y}(X^{OA})/X^{OA} = c$, that is, total revenues and total costs are equal such that rents are exhausted.

Figure 4 illustrates a case for which the open-access equilibrium yields a higher sustainable harvest rate than the exclusive one, which may give the impression that the open-access equilibrium is preferable to the exclusive one. This is clearly not the case as the marginal product of effort with free-access falls short of its cost. To convince yourself of this, note that in the illustrated case, the marginal product of effort being negative implies that steady-state output could be increased with a lower effort. But although exclusive access per-period profit maximization will always result in a positive marginal product with a stock size larger than that of the maximum sustainable yield, the open-access equilibrium may fall on either side; whether this is the case or not depends on where the total cost and total revenues curves meet each other.

0.1.5 Open access and the backward-bending supply curve

In section 0.1.4, we learned that for a *given* output price, the steady-state production level varies with the property regime. We now wish to extend the analysis at the industry level where the price is made endogenous with demand and supply schedules. To this end, we shall first build the supply schedules in both open access and exclusive access. We then posit a demand schedule and look at how variations in the property regime can lead to quite different market equilibria when combined with the resource's biological dynamics.[4]

In figure 5, industry supply curves are illustrated in panel b) for exclusive access and open access, denoted S_{EX} and S_{OA} respectively. Each is built with the help of panel a). In panel a), three steady state total revenue curves have been drawn which correspond to three different given price levels: $p_A < p_B < p_C$. The revenue curves are thus labeled $p_A y^{ss}$, $p_B y^{ss}$ and $p_C^y ss$ and have been drawn according to the yield-effort curve in figure 3. For convenience, we assume that $p_B = 1$; this allows us to locate the quantities supplied y^{ss} on curve $p_B y^{ss}$ and report it in panel b). The total cost curve TC illustrated in panel a) is independent of output prices and access regimes.

[4]The analysis is based on Copes (1970).

Let us begin by building the open access supply curve S_{OA} in panel b). In this case, total rents must be completely dissipated, such that the supply equilibria are represented by points A, B and C in panel a) for the three different price levels. Given that $p_B = 1$, the steady-state output levels which correspond to the respective equilibrium input levels X_A, X_B and X_C are located on curve $p_B y^{ss}$; it is then used to locate three corresponding points A, B and C on the supply curve in panel b). The rest of the open-access supply curve can be built using this procedure by assuming different price levels in panel a). It results in supply schedule S_{OA}.

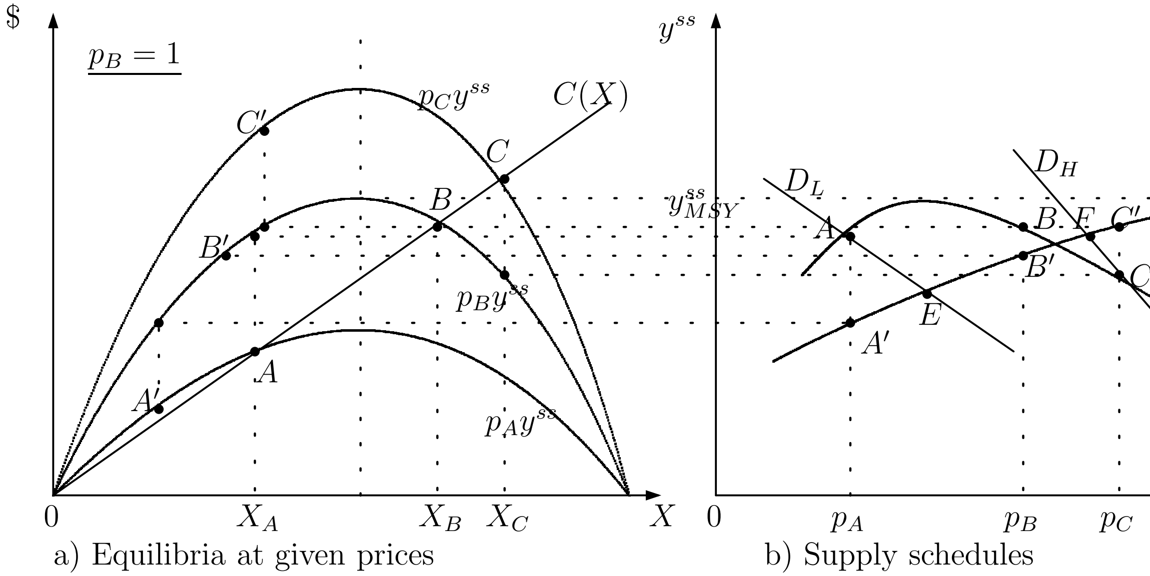

Figure 5: Property regime and industry equilibrium

We note that the supply curve in open access is *backward bending*. At sufficiently low output price levels, the steady-state resource stock level is larger than the maximum sustainable yield stock (S_{MSY}) size. Consequently, the higher input effort level induced by a higher output price causes the supply to increase. On the other hand, when the output price is already high, the open access equilibrium leads to a steady-state stock level which is smaller than S_{MSY} and consequently, any increased effort induced by a higher output price causes the output to fall in steady state. Due to the reproductive properties of the resource, we therefore obtain a *backward-bending supply curve* in open access.

Turning now to the supply schedule under exclusive access, the supply equilibria corresponding to prices p_A, p_B and p_C are respectively given by points A', B' and C' in panel a), i.e. where the slope of the total revenue is equal to the slope of the total cost curve. (The reader is encouraged to verify that the input effort must increase with the price level.) The output levels that corresponds to those equilibria are again found on curve $p_B y^{ss}$ and are in panel b) at points A', B' and C'. Proceeding this way with different price levels, the exclusive access supply schedule S_{EX} can be built.

We note here that under exclusive access, the supply schedule is always positively sloped. Looking at panel a), this result can be deduced from the fact that with a positively sloped total cost curve, the equilibrium supply must be determined by a point on the positively sloped portion of the total revenue curve. Consequently, the stock size of the resource always lies above S_{MSY} under exclusive access, with the result that an increased input effort causes the output to increase.[5]

We therefore obtain that when the demand schedule is low, as illustrated by curve D_L in panel b), the market equilibrium under open access (point A) is characterized by a higher output and lower price than under exclusive access (point E). But the converse holds when the demand schedule is high, as illustrated by curve D_H: the open access equilibrium (point C) is associated with a lower output level and higher price than under exclusive access (point F). Comparing the open access equilibrium points A and C, we further obtain the paradoxical result that an increase in demand leads to a lower equilibrium output level.

[5]We shall see below, however, that with a positive discount rate, the steady-state stock size of the resource may be smaller than S_{MSY} under exclusive access also.