0.1 A first example: Land and locational rents

Suppose that agricultural production requires only land an labour as inputs.[1] Let y denote the total output from a particular plot of land, in units of wheat bushels. The technology of production is described by total output function $y = f(x)$, where x is the labour input quantity, say in hours. Function f is increasing and concave. In figure 1, the output technology is conveniently summarized by both the average product of labour $\phi(x) \equiv f(x)/x$ and its marginal product $f'(x)$.

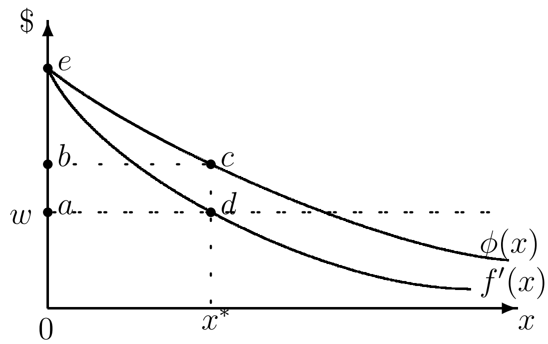

Figure 1: Output and rent on a single plot of land

We assume that the hourly labour wage is constant and equal to w in units of wheat bushels. The profit maximizing labour input is thus given by quantity x^* such that $f'(x^*) = w$. This yields a profit equal to square area $\diamond abcd$,[2] that is, the difference between the average product and the average wage wage multiplied by the total hours worked. For future analysis, it will be useful to note that profit can also be represented by the area under the marginal product curve minus the wage bill, that is, area $\diamond aed$. We shall refer to this profit as a rent because it can be achieved by one particular plot of land which cannot be replicated. To see why, let us consider two plots of land, each located at a different distance from the market.

Suppose that plots A and B are identical except for the fact that plot A is located closer to the market than plot B. This translates into a higher "gate

[1] The analysis in this section is originally attributed to von Thunen (xxxx).

[2] We use the symbol $\diamond$ to denote a surface.

price" for A's output due to lower transport costs. Let p_A and p_B denote the gate price per wheat bushel obtained at plots A and B respectively, with $p_A > p_B$. We add the new twist that labour hours are inelastically supplied at a maximum of $\bar{x}$, that is, $x_A + x_B \leq \bar{x}$. All prices are expressed in terms of a currency units called the *sol* and represented by the symbol \$.

Exercise 1 **Analyze the case where the total supply of labor is perfectly elastic at fixed wage w.*

The values of labour's marginal products on each plot are illustrated in figure 2. The labour input on plot A increases as we move rightward from origin 0_A while the same goes for plot B moving leftward from origin 0_B, where segment length $\overline{0_A 0_B} = \bar{x}$.[3] Note that for any given x, the value of labour's marginal product is higher on plot A than plot B simply because $p_A > p_B$.

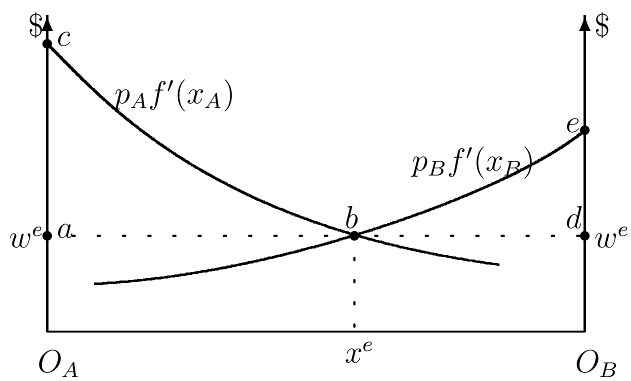

Figure 2: Locational rents with fixed labor supply

The efficient allocation of labour between both plots is given by x^e, which equates the marginal productivity values. It corresponds to labour quantities $\overline{O_A x^e}$ and $\overline{0_B x^e}$ working on plots A and B respectively. *Note that we purposefully stay silent on how this distribution of workers is achieved in practice.* With such a distribution, the profits on each plots are respectively given by $R_A = \diamond abc$ and $R_B = \diamond dbe$.

[3]A bar over coordinate points shall denote a segment length.

With the efficient allocation, we have $R_A > R_B$. Plot A generates higher rents because it commands a higher gate price. This is due to the immovable fact that it is located closer to the market than plot B *and* that it cannot be replicated. It is in this respect that we say that plot A yields higher rents than plot B. In the example given, such rents are referred to as *locational* rents, as first pointed out by von Thunen (xxxx).

If we were to add a third plot C with unit bushel gate price $p_C < p_B$, the efficient equilibrium allocation of labour would be given by

$$p_A f'(x_A^e) = p_B f'(x_B^e) = p_C f'(x_C^e), \tag{1}$$
$$x_A^e + x_B^e + x_C^e = \bar{x}, \tag{2}$$

as illustrated in figure 3.

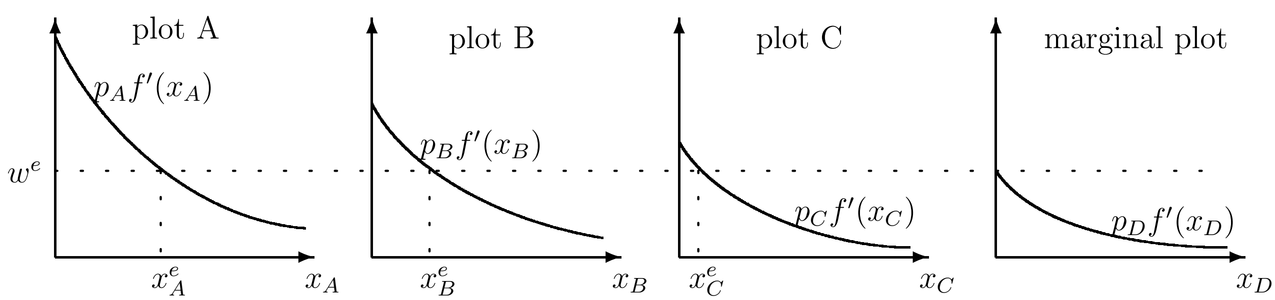

Figure 3: Locational rents and marginal plots

For a far enough distance from the market, one may eventually find that the gate price of, say, plot E is so low that its marginal product yields $p_E f'(0) < p_C f(x_C^e)$ while equations (1) and (2) remain respected. Plot E is therefore located too far away from the market to make it efficient to exploit. The upshot is to imagine a situation with a large number of land plots, each located farther and farther away from the market center. Efficiency then dictates an allocation of workers such that at a certain distance from the market, say at plot D, we have $p_D f'(0) = w^e$, where w^e is the efficient marginal productivity of labour on the plots being used. All plots located closer to the market are characterized by a gate price larger than p_D, are put to use, and generate positive rents that decrease with the distance. All those located farther away have a lower gate price than p_D and are left unused. In the literature, plot D is referred to as the *marginal land*.

0.2 A second example: Non-renewable resources and dynamic rents

We now look at a basic two-period problem for the extraction of a non-renewable resource. We will see that due to the finite size of the resource stock, a present-value maximizer will leave some rents to be extracted later.

Let S_0 denote the initial stock size of the resource which can be extracted over two periods, represented by $t = 0$ and $t = 1$. A unit of the resource sells at constant price p at each period. The cost of extraction at period t depends on the extraction rate only and is represented by function $C(R_t)$, where R_t is the period-t extraction level and function C is increasing and convex. Assuming that period-1 gains are discounted at factor β into period 0, the present-value maximizing problem can be expressed as

$$\max_{R_0,R_1} V_0 = pR_0 - C(R_0) + \beta[pR_1 - C(R_1)] \tag{3}$$

$$\text{s.t.} \quad R_0 + R_1 \leq S_0 \tag{4}$$

Assuming that the resource constraint is binding, we can conveniently substitute $R_1 = S_0 - R_0$ into the objective function. This yields the following first-order condition for a maximum:

$$\frac{\partial V_0}{\partial R_0} = p - C'(R_0^*) + \beta[-p + C'(R_1^*)] = 0. \tag{5}$$

This expression indicates that the contribution to present-value V_0 of the last unit extracted at periods 0 and 1 must be equal. If this were not the case, say with inequality $p - C'(R_0) > \beta[-p + C'(R_1)]$, then V_0 could be increased by extracting one more unit at period 0 and one less at period 1, and conversely if the inequality is reversed. $p - C'(R_t)$ is referred to as the *marginal rent* at period t because it concerns only the rent on the last unit extracted.[4]

The marginal rent curves for each periods are illustrated in figure 4, where the length of the abscissa is equal to the initial resource stock S_0 and the

[4]It is also sometimes called *dynamic rent*, *user cost*, *royalty*, *Hotelling rent*, or *in situ rent*.

origin for periods 0 and 1 extraction rates are respectively denoted 0_0 and 0_1. Note that the height of the marginal rent curves are expressed in present value terms, which explains with its height at period 1 is lower than at period 0. In this example, the fact that the whole resource stock can be extracted at both periods while marginal rents remain positive indicates that the resource constraint must be binding; indeed, the last unit extracted contributes strictly positively to the present value of the resource stock V_0.

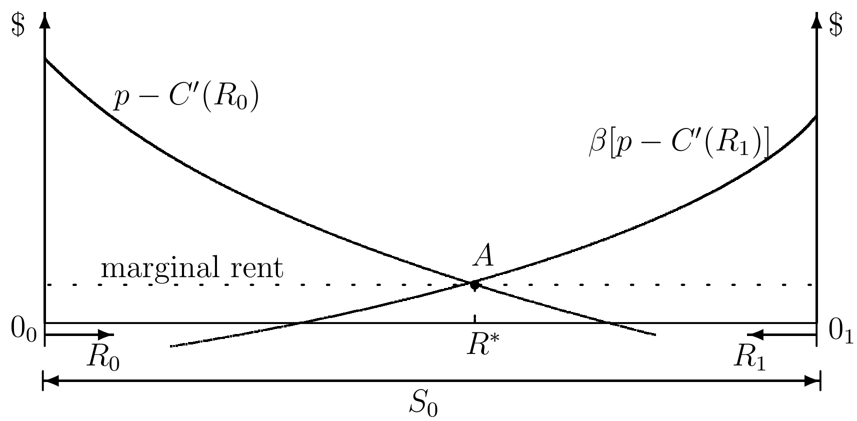

Figure 4: Two-period non-renewable resource extraction

At this stage, students are sometimes puzzled by the fact that an optimal use of the resource does not dictate that the price be equal to the marginal cost of extraction. This is so here is because with a binding resource constraint, the last unit extracted at period 0 is really being taken away from period 1. Consequently, the total opportunity cost of extraction at period 0 must incorporate the forgone gain in period 1, i.e., $p - C'(R_1)$. Using the fact that future gains are discounted at factor β, the marginal opportunity cost of extraction at period 0 is equal to $C'(R_0) + \beta[p - C'(R_1)]$ and equality (5) indeed insures that this is equal to the price. This explains why the the optimal price is above the direct marginal cost of extraction $C'(R_0)$.

The upshot is that due to a non-renewable resource's limited size, its optimal use will generally dictate that some present rents be left into the ground for future benefits. This is another instance of a *scarcity rent*.

It is important, however, to keep in mind that period 0's marginal rent value is endogenous, i.e., *its value is determined by the solution to the present-*

value maximizing problem. Indeed, if, for whatever reason, R_0 had been chosen such that $p = C'(R_0)$, the marginal rent would be zero. This is a fundamental point because as an owner seeks to maximize the present value of a resource by leaving rents into the ground, tantamount to an investment decision, one implicitly assumes that she is totally confident that she will be in a position to reap all the rewards from that investment. Things are usually not so clearcut in real life. This is exemplified by problem **??** below, in which a simple change in the institutional setting is shown to drastically alter the owner's equilibrium marginal rent.