2006 Mid-term Question 7[1]

I can think of two ways to approach this question. One is by starting from the profit maximization problem using the cost function, the other is with the supply function. I will do both.

1) Solution based on the cost function:

$$\pi = py - c(\mathbf{w}, y) \tag{1}$$

The FOC for profit max. is:

$$\frac{\partial \pi}{\partial y} = p - \frac{\partial}{\partial y} c(\mathbf{w}, y) = 0, \tag{2}$$

$$\text{or } \frac{\partial}{\partial y} c(\mathbf{w}, y) = p \ \text{ (a constant)}. \tag{3}$$

QUESTION: What are the necessary SOC for this solution to hold? Would a CRS technology be admissible here?

Relation 2 gives us an implicit relation between w_1, w_2 and y. As far as we are concerned here, it says that if w_1 and w_2 both increase, the firm will adjust y in order to keep $\frac{\partial}{\partial y} c(\mathbf{w}, y)$ constant and equal to p. Hence the following:

$$\frac{\partial}{\partial w_1} \frac{\partial}{\partial y} c(\mathbf{w}, y) dw_1 + \frac{\partial}{\partial w_2} \frac{\partial}{\partial y} c(\mathbf{w}, y) dw_2 + \frac{\partial^2}{(\partial y)^2} c(\mathbf{w}, y) dy = 0. \tag{4}$$

Using Shephard's Lemma and rearranging, we get:

$$dy = -\frac{\frac{\partial}{\partial y} \left[x_1(\mathbf{w}, y) dw_1 + x_2(\mathbf{w}, y) dw_2 \right]}{\frac{\partial^2}{(\partial y)^2} c(\mathbf{w}, y)}. \tag{5}$$

By the SOC, the denominator is positive. Hence

$$dy > 0 \text{ iff } \frac{\partial}{\partial y} x_1(\mathbf{w}, y) dw_1 + \frac{\partial}{\partial y} x_2(\mathbf{w}, y) dw_2 < 0. \tag{6}$$

This implies that one of the two input types must be an *inferior factor*. It means that in order to produce more output, the firm actually <u>chooses</u> to use less of that input. We have shown that this leads to the paradoxical possibility that the output increases even though

[1] NB This question was virtually identical to assigned exercise no 5.10.

the price of both inputs has increased.

QUESTION: Doesn't this result contradict the fact that own-price effects are nonpositive? Explain.

QUESTION: Provide a graphical example of an inferior factor.

QUESTION: Can both factors be inferior?

2) Solution based on supply function:

Take the supply function $y(\mathbf{w}, p)$. Since w_1 and w_2 both increase, we have, by definition,

$$dy = \frac{\partial y(\mathbf{w}, p)}{\partial w_1} dw_1 + \frac{\partial y(\mathbf{w}, p)}{\partial w_2} dw_2. \tag{7}$$

By symmetry of the substitution matrix, we get

$$dy = -\frac{\partial x_1(\mathbf{w}, p)}{\partial p} dw_1 - \frac{\partial x_2(\mathbf{w}, p)}{\partial p} dw_2. \tag{8}$$

Since $x_i(\mathbf{w}, p) = x_i(\mathbf{w}, y(\mathbf{w}, p))$, we have

$$dy = -\frac{\partial x_1(\mathbf{w}, y)}{\partial y} \frac{\partial y(\mathbf{w}, p)}{\partial p} dw_1 - \frac{\partial x_2(\mathbf{w}, y)}{\partial y} \frac{\partial y(\mathbf{w}, p)}{\partial p} dw_2 \tag{9}$$

Hence, we obtain the same conclusion as in (6).