August 4, 2015

ECO 6122: Microeconomic Theory IV
Economics Department
University of Ottawa

final exam
Time allotted: 3 hours
Professor: Louis Hotte

3. (25 points) Nash equilibrium and price competition

Firm A and firm B are two textile companies that produce the same silk cloth. Firm A has a technology that uses only labor (L) as an input. Its production function is

$$Q_A = L.$$

Firm B has a technology that uses both capital (K) and labor (L). Its production function is

$$Q_B(K, L) = 2\sqrt{KL}$$

. Let r and w represent the capital rental rate and the wage rate, and assume that both companies are price-takers in the input markets.

Suppose that the market demand for silk cloth is given by

$$Q = 120 - P$$

where P is the prevailing market price of cloth.

Both firms are profit maximizers. The two firms set the prices of their cloth (P_A and P_B) simultaneously, where P_A and P_B can take any integer value between 0 and 120 (no fractional prices are permitted). If $P_A \neq P_B$, consumers buy only from the lower-price firm; if $P_A = P_B$ they divide their purchases equally between the two firms.

Find Nash equilibrium prices when $w = 10$ and $r = 40$. Explain briefly but clearly all the steps in your work.

REMARK: This problem constitutes a nice way to combine various important concepts seen in this course and apply them to a new setting. It calls for the concepts of cost minimization, conditional input demands, Nash equilibrium and Bertrand competition.

SOLUTION

Since firms choose prices, let us express their profit as a function of prices.

The payoff function for firm A is $\pi_A = P_A Q_A - wL_A$ where $Q_A = L_A$. Hence $\pi_A = (P_A - w)Q_A$, where Q_A depends on both prices P_A and P_B as follows:

$$
\begin{aligned}
& && 120 - P_A \text{ if } P_A < P_B \\
Q_A &=&& (120 - P_A)/2 \text{ if } P_A = P_B \\
& && 0 \text{ if } P_A > P_B
\end{aligned}
$$

Note that the profit function is discontinuous in prices. Hence, one cannot take the derivative of profits w.r.t. the price in order to find the FOC.

For firm B, the profit is given by $\pi_B = P_B Q_B - wL_B - rK_B$, where Q_B depends on P_A and P_B analogously to firm A. In order to express profits as a function of P_B only, let us find the cost minimizing way of producing any output level Q_B. The problem is as follows:

$$\min_{L_B, K_B} c(w, r, Q_B) = wL_B + rK_B \text{ s.t. } Q_B = 2\sqrt{K_B L_B}.$$

Taking the Lagrangian, it is easy to verify that the FOCs give $K_B/L_B = w/r$. Through substitution into the production function, we get the following conditional input demands: $L_B^* = (1/2)\sqrt{r/w}Q_B$ and $K_B^* = (1/2)\sqrt{w/r}Q_B$. Hence, we have:

$$
\begin{aligned}
& c(w, r, Q_B) = (1/2)\sqrt{rw}Q_B, \\
& \pi_B = (P_B - (1/2)\sqrt{rw})Q_B.
\end{aligned}
$$

Using the fact that $w = 10$ and $r = 40$, we have:

$$
\begin{aligned}
& \pi_A = (P_A - 10)Q_A \\
& \pi_B = (P_B - 10)Q_B
\end{aligned}
$$

We note that both firms actually have the same constant marginal cost of production. The problem is thus identical to the Bertrand competition model analysed in class, with the difference that we only allow for integer prices. But it is easy to verify that just like the standard Bertrand model, marginal cost pricing is a Nash equilibrium.

Let $P_A^* = P_B^* = 10$. Then $\pi_A^* = \pi_B^* = 0$. Set $P_B = 10$. If firm A asks for a price $P_A > 10$, its profits are nil since $Q_A = 0$. And if $P_A < 10$, then $\pi_A < 0$. Hence firm A cannot gain by deviating from $P_A = 10$. And similarly for firm B.

Due to integer prices – and contrary to the Bertrand model seen in class – we can similarly verify that $P_A^* = P_B^* = 11$ is a Nash equilibrium. (Though the question did not ask to find all Nash equilibria.) Equilibrium profits are strictly positive in this case.