MICRO IV
MID-TERM FALL 2008
SOLUTIONS.

A.1) $e(\vec{p},u)$ is homo. of deg. 1 in $\vec{p}$?
TRUE.

DEMO: We have $e(\vec{p},u) = \vec{p}x(\vec{p},u)$
and $e(t\vec{p},u) = t\vec{p}x(t\vec{p},u)$.

We must show that $e(t\vec{p},u) = t\,e(\vec{p},u)$.
Assume not. Then, by expenditure minimization, we must have

$$t\vec{p}x(t\vec{p},u) < t\vec{p}x(\vec{p},u).$$

$$\Rightarrow \vec{p}x(t\vec{p},u) < \vec{p}x(\vec{p},u).$$

This implies that $x(\vec{p},u)$ does not minimize expenditure at $\vec{p}$.

A contradiction.

$\Rightarrow e(t\vec{p},u) = t\,e(\vec{p},u)$ QED.

A.2) $c(\vec{w}, y) = A w_1^{\alpha} w_2^{\beta} y$

a) $x_1(\vec{w}, y) = \frac{\partial c}{\partial w_1}$ by Shephard's lemma.

$$= \alpha A w_1^{\alpha-1} w_2^{\beta} y$$

$$= \frac{\alpha c(\vec{w}, y)}{w_1}$$

Similarly for $x_2(\vec{w}, y)$.

b) $s_1 = w_1 x_1 / c$

$$= \frac{\alpha A w_1^{\alpha} w_2^{\beta} y}{A w_1^{\alpha} w_2^{\beta} y} = \alpha.$$ QED

Similarly for s_2.

A.3) $$\eta_i = \frac{\partial x_i}{x_i} \Big/ \frac{\partial y}{y}$$

By budget balancedness, we have

$$\sum_i p_i \cdot x_i = y$$

$$\Rightarrow \frac{\partial}{\partial y} \sum_i p_i \cdot x_i = 1$$

$$\Rightarrow \sum_i \left[p_i \frac{\partial x_i}{\partial y} \right] = 1$$

$$\Rightarrow \sum_i \left[p_i \frac{\partial x_i}{\partial y} \cdot \frac{y}{x_i} \cdot \frac{x_i}{y} \right] = 1$$

$$\Rightarrow \sum_i \left[\frac{p_i x_i}{y} \; \frac{\partial x_i}{x_i} \Big/ \frac{\partial y}{y} \right] = 1$$

$$\Rightarrow \sum_i s_i \eta_i = 1.$$ <u>QED</u>

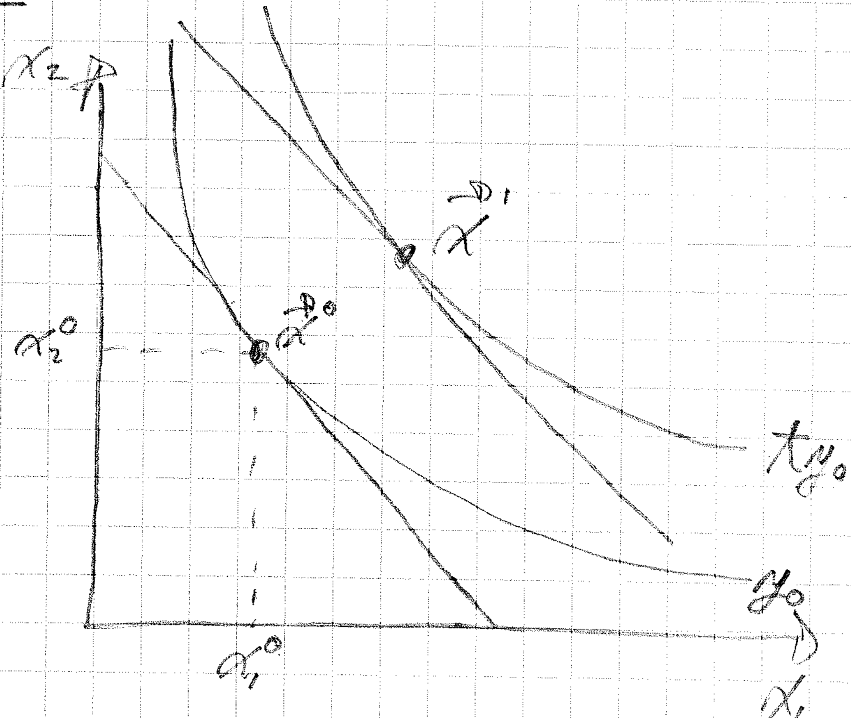
B.1 Homo. of degree 0 implies: B1

$$\vec{x}(\vec{w}, y) = \vec{x}(t\vec{w}, ty).$$

This is FALSE.

$\vec{x}^0 = \vec{x}(\vec{w}, y^0)$

$\vec{x}^1 = \vec{x}(t\vec{w}, ty^0)$



If $t > 1$, then we have $ty^0 > y^0$ while the slope of the isocost curve is unchanged. In this example, we get $\vec{x}(t\vec{w}, ty^0) > \vec{x}(\vec{w}, y^0)$. Hence the conditional demand is not homo. of deg. 0 in $(\vec{w}, y)$. The bottom line is that you cannot produce more output with the same input vector.

B.2

a) By budget balancedness, we
3 have $p_1x_1 + p_2x_2 + p_3x_3 = y$.

$$\Rightarrow x_3(\vec{p}, y) = \frac{y - p_1x_1 - p_2x_2}{p_3}$$

All is left to do is to substitute $x_1(\vec{p}, y)$ and $x_2(\vec{p}, y)$ to get $x_3(\vec{p}, y)$.

b) The Marshallian demands should
3 be homogeneous of degree 0 in $(\vec{p}, y)$, i.e. $x_i(t\vec{p}, ty) = x_i(\vec{p}, y)$.

We have

$$x_1(t\vec{p}, ty) = 100 - 5\frac{tp_1}{tp_3} + \beta\frac{tp_2}{tp_3} + \delta\frac{ty}{tp_3}$$

$$= 100 - 5\frac{p_1}{p_3} + \beta\frac{p_2}{p_3} + \delta\frac{y}{p_3}$$

$$= x_1(\vec{p}, y) \qquad \underline{QED}$$

And similarly for $x_2(\vec{p}, y)$.

4 c) By <u>symmetry</u> of the Slutsky matrix, we have:

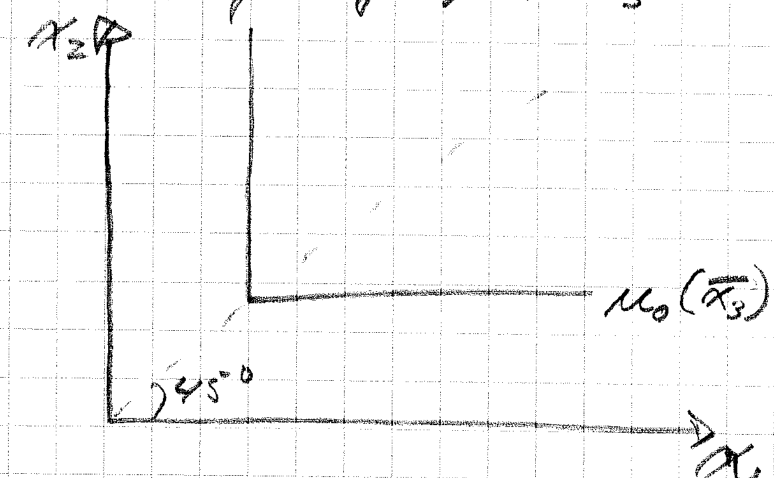
$$\frac{\partial x_1}{\partial p_2} + x_2\frac{\partial x_1}{\partial y} = \frac{\partial x_2}{\partial p_1} + x_1\frac{\partial x_2}{\partial y}$$

$$\Rightarrow \frac{\beta}{p_3} + x_2\frac{\delta}{p_3} = \frac{\beta}{p_3} + x_1\frac{\delta}{p_3} \Rightarrow x_2 = x_1$$

$$\Rightarrow 100 - 5\frac{p_1}{p_3} + \beta\frac{p_2}{p_3} = \alpha + \beta\frac{p_1}{p_3} + \gamma\frac{p_2}{p_3}$$

$$\Rightarrow \alpha = 100,\ \beta = -5 = \gamma.$$ QED

3 d) Since $x_1 = x_2$, we have a Leontief utility function for fixed x_3:



$$\Rightarrow u(x_1, x_2, \overline{x}_3) = u(\min\{x_1, x_2\}, \overline{x}_3)$$

4 e) By negative semi-definiteness of the Slutsky matrix, we have

$$\frac{\partial x_1}{\partial p_1} + x_1\frac{\partial x_1}{\partial y} \leq 0.$$

$$\Rightarrow -\frac{5}{p_3} + x_1\frac{\delta}{p_3} \leq 0$$

$$\Rightarrow \delta x_1 \leq 5$$

$$\Rightarrow \delta\left(100 - 5\frac{p_1}{p_3} - 5\frac{p_2}{p_3} + \delta\frac{y}{p_3}\right) \leq 5$$

Since y is unbounded above, we can always find y large enough such that this inequality is violated when $\delta^2 > 0$.

$$\Rightarrow \delta = 0.$$ QED

f) With $\delta = 0$, we have $\frac{\partial x_1}{\partial y} = \frac{\partial x_2}{\partial y} = 0$.

3 By budget balancedness, we thus have

$$\frac{\partial x_3}{\partial y} = 1 . \qquad \left(\sum_i \frac{\partial x_i}{\partial y} = 1 \right)$$

The utility fctn must respect the following:

① $x_1 = x_2$

② All additional income is spent on good 3.

③ $\frac{\partial x_1}{\partial p_1} = \frac{\partial x_2}{\partial p_2} = -5$ (If $p_3 = 1$ i.e. good 3 is the numeraire)

④ $\lim_{p_1 + p_2 \to 0} x_i = 100$, $i = 1, 2$

⑤ If $(p_1 + p_2)/p_3 \geq 20$, then $x_1 = x_2 = 0$.

$$u = f(\min\{x_1, x_2\}, x_3)$$