ECO 6122
FINAL EXAM 2008.

#3) a) If both farmers have cooperated in the past, continued cooperation has the following present value for farmer 1:

$$V_1 = \pi^m + \beta\pi^m + \beta^2\pi^m + \ldots$$

where $\beta = \frac{1}{1+r}$.

$$\Rightarrow V_1 = \frac{\pi^m}{1-\beta} = \frac{1+r}{r}\pi^m$$

If farmer 1 decides to deviate from the trigger strategy instead, he gets an one-shot profit of π^d. After that, profits fall to zero due to the trigger strategy being played by farmer 2. The present value of such a deviation is thus:

$$V_1' = \pi^d + \beta 0 + \beta^2 0 + \ldots = \pi^d$$

Cooperation can thus be sustained if $V_1 \geq V_1'$, or

$$\frac{\pi^m}{1-\beta} \geq \pi^d \Rightarrow \frac{\pi^m}{\pi^d} \geq 1-\beta = \frac{r}{1+r}$$

$$\Rightarrow \pi^m + r\pi^m \geq r\pi^d \Rightarrow \boxed{\frac{\pi^m}{\pi^d - \pi^m} \geq r}$$

The maximum value of the discount rate is thus $\frac{\pi^m}{\pi^d - \pi^m}$ for cooperation to be sustained.

b) If farmers only meet every four weeks, the discount factor between periods of interaction is now β^4 instead of β. We have:

$$V_1 = \pi^m + \beta^4 \pi^m + \beta^8 \pi^m + \ldots$$

$$= \frac{1}{1-\beta^4}\pi^m$$

$$V_1' = \pi^d$$

$$\Rightarrow V_1 \geq V_1' \text{ if } \frac{\pi^m}{1-\beta^4} > \pi^d \Rightarrow \left(\frac{1}{1+r}\right)^4 \geq 1 - \frac{\pi^m}{\pi^d}.$$

When farmers met weekly, cooperation could be sustained with:

$$\frac{1}{1+r} \geq 1 - \frac{\pi^m}{\pi^d}.$$

Since $\left(\frac{1}{1+r}\right)^4 < \frac{1}{1+r}$, the new town bylaw makes cooperation more difficult to sustain between farmers.

c) The present value of cooperation is now:

$$V_1 = \pi^m + \beta(1+g)\pi^m + \beta^2(1+g)^2\pi^m + \ldots$$

$$= \frac{\pi^m}{1-(1+g)\beta} = \frac{1+r}{r-g}\pi \quad \text{(N.B. Assume } g<r.)$$

$$V_1' = \pi^d + \beta 0 + \beta^2 0 + \ldots = \pi^d$$

Sustained cooperation requires

$$V_1 \geq V_1' \Rightarrow (1+g)\left(\frac{1}{1+r}\right) \geq 1 - \frac{\pi^m}{\pi^d}$$

Since $(1+g)\left(\frac{1}{1+r}\right) > \frac{1}{1+r}$ for $g > 0$,

cooperation becomes <u>easier</u> to sustain when demand is growing.

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#4)
$$u(\vec{x}) = (x_1 - a_1)^\alpha x_2^{1-\alpha}$$

a) Indirect utility fctn

$$\max_{\vec{x}} u(\vec{x}) \text{ s.t. } \vec{p}\cdot\vec{x} \le y$$

$$\mathcal{L} = (x_1-a_1)^\alpha x_2^{1-\alpha} + \lambda\left(y - \sum_{i=1}^{2} p_i x_i\right)$$

$$\frac{\partial \mathcal{L}}{\partial x_1} = \alpha(x_1-a_1)^{\alpha-1} x_2^{1-\alpha} - \lambda p_1 = 0. \qquad [A1]$$

$$\frac{\partial \mathcal{L}}{\partial x_2} = (1-\alpha)(x_1-a_1)^\alpha x_2^{-\alpha} - \lambda p_2 = 0. \qquad [A2]$$

$$\left.\begin{array}{l} [A1] \Rightarrow \dfrac{\alpha u(\vec{x})}{x_1 - a_1} = \lambda p_1 \\ [A2] \Rightarrow \dfrac{(1-\alpha) u(\vec{x})}{x_2} = \lambda p_2 \end{array}\right\} \Rightarrow \frac{\alpha}{p_1(x_1-a_1)} = \frac{1-\alpha}{p_2 x_2}$$

$$\Rightarrow x_2 = \frac{(1-\alpha)p_1(x_1-a_1)}{\alpha p_2}$$

$$\Rightarrow p_2 x_2 = \frac{(1-\alpha)}{\alpha} p_1(x_1-a_1)$$

$$\Rightarrow y = p_1 x_1 + \frac{(1-\alpha)}{\alpha} p_1(x_1-a_1)$$

$$\Rightarrow \alpha y = \alpha p_1 x_1 + (1-\alpha)p_1 x_1 - (1-\alpha)p_1 a_1$$

$$\Rightarrow x_1 = \frac{\alpha y + (1-\alpha)p_1 a_1}{p_1} = \alpha\frac{y}{p_1} + (1-\alpha)a_1$$

$$\Rightarrow x_1 - a_1 = \alpha\frac{y}{p_1} - \alpha a_1 = \alpha\left(\frac{y}{p_1} - a_1\right) = \frac{\alpha}{p_1}(y - p_1 a_1)$$

$$\Rightarrow x_2 = (1-\alpha)\frac{p_1}{p_2}\left(\frac{y}{p_1} - a_1\right) = \frac{(1-\alpha)}{p_2}(y - p_1 a_1)$$

$$\Rightarrow v(\vec{p}, y) = \left(\frac{\alpha}{p_1}\right)^\alpha (y - p_1 a_1)^\alpha \left(\frac{1-\alpha}{p_2}\right)^{1-\alpha} (y - p_1 a_1)^{1-\alpha}$$

$$\boxed{v(\vec{p}, y) = \left(\frac{\alpha}{p_1}\right)^\alpha \left(\frac{1-\alpha}{p_2}\right)^{1-\alpha} (y - p_1 a_1)}$$

✓

b) <u>Expenditure fctn</u>

$$u = v(\vec{p}, e(\vec{p}, u))$$

$$\Rightarrow u = \left(\frac{\alpha}{p_1}\right)^{\alpha}\left(\frac{1-\alpha}{p_2}\right)^{1-\alpha}\left(e(\vec{p}, u) - p_1 a_1\right)$$

$$\Rightarrow \boxed{e(\vec{p}, u) = \left(\frac{p_1}{\alpha}\right)^{\alpha}\left(\frac{p_2}{1-\alpha}\right)^{1-\alpha} u + p_1 a_1}$$ ✓

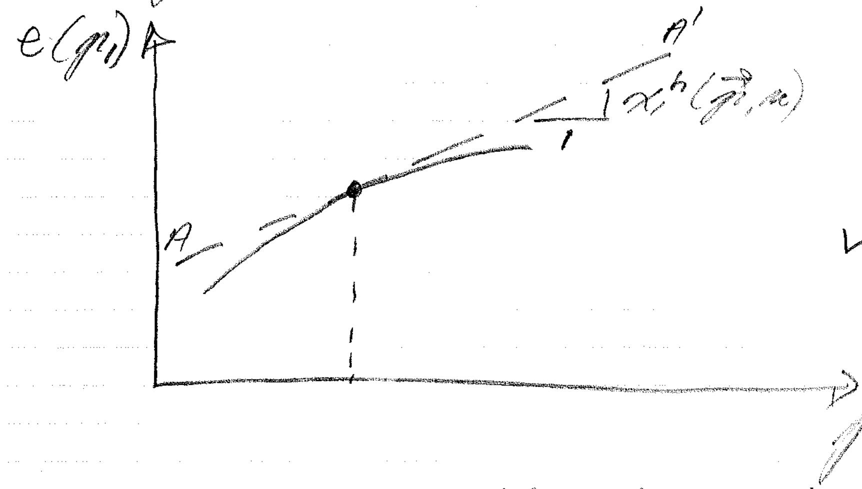
c) The expenditure fctn should be concave with respect to p_1. This can be seen from the fact that it is linear in $\vec{x}$:

$$e(\vec{p}, u) = p_1 x_1^h(\vec{p}, u) + p_2 x_2^h(\vec{p}, u)$$

where x_i^h is Hicksian demand.

If p_1 varies while consumption remains unchanged, we have

$\frac{\partial e}{\partial p_1} = x_1^h(\vec{p}, u)$.



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By changing its consumption bundle as p_1 varies, the minimum expenditure necessary to achive utility level u cannot be above straight line AA'. $e(\vec{p}, u)$ is thus concave in p_1.

Indeed, from part b) we have:

$$\frac{\partial e}{\partial p_1} = \alpha \frac{p_1^{\alpha-1}}{\alpha^\alpha}\left(\frac{p_2}{1-\alpha}\right)^{1-\alpha} u + a_1$$

$$\frac{\partial^2 e}{\partial p_1^2} = \alpha(\alpha-1) \frac{p_1^{\alpha-2}}{\alpha^\alpha}\left(\frac{p_2}{1-\alpha}\right)^{1-\alpha} u < 0. \checkmark$$