

$$\#3 \quad v(\vec{p}, y) = \frac{y}{p_1 + p_2}$$

a)  $\alpha_i = - \frac{\frac{\partial v}{\partial p_i}}{\frac{\partial v}{\partial y}}$  ; ordinary demand  
by Roy's identity

$$\frac{\partial v}{\partial p_1} = \frac{-y}{(p_1 + p_2)^2} \quad \frac{\partial v}{\partial y} = \frac{1}{p_1 + p_2}$$

$$\alpha_1(\vec{p}, y) = \frac{y}{(p_1 + p_2)^2} = \alpha_2(\vec{p}, y)$$

b) We know that the following equality must hold:

$$v(p, e(p, u)) = u$$

$$\Rightarrow \frac{e(p, u)}{p_1 + p_2} = u \Rightarrow e(p, u) = u(p_1 + p_2)$$

c) Direct utility:

(N.B. This was a harder question for someone who has never seen a Leontief production function. I was looking for a reasoning.)

We see that  $\alpha_1(\vec{p}, y) = \alpha_2(\vec{p}, y)$  regardless of relative prices. This suggests no possibility of substitution between the two goods. The function that represents this is:

$u = \min\{\alpha_1, \alpha_2\}$  or, graphically:

