MICRO IV

SOLUTIONS MID-TERM I 2015W

#1 The expenditure function:

a) Definition: The expenditure function is the function $e(\vec{p}, u)$ that minimizes the expenditure necessary to attain a given utility level u at given prices $\vec{p}$. Formally:

$$e(\vec{p}, u) = \min_{\vec{x}} \vec{p}\vec{x} \text{ s.t. } u(\vec{x}) \geq u.$$

b) Concavity in prices of $e(\vec{p}, u)$.

Show that:

$e(\vec{p}^t, u) \geq t\,e(\vec{p}^1, u) + (1-t)\,e(\vec{p}^2, u)$, $\forall t \in [0,1]$

where $\vec{p}^t = t\vec{p}^1 + (1-t)\vec{p}^2$.

PROOF:

If $x(\vec{p}^1, u)$ and $x(\vec{p}^2, u)$ are bundles that minimize the cost of attaining utility level u, then the following must hold:

$$\vec{p}^1\vec{x}(\vec{p}^1, u) \leq \vec{p}^1\vec{x}(\vec{p}^t, u) \quad [A]$$

$$\vec{p}^2\vec{x}(\vec{p}^2, u) \leq \vec{p}^2\vec{x}(\vec{p}^t, u) \quad [B]$$

We consequently have:

$$t\vec{p}^{\,1}\vec{x}(\vec{p}^{\,1};u)+(1-t)\vec{p}^{\,2}\vec{x}(\vec{p}^{\,2};u)$$

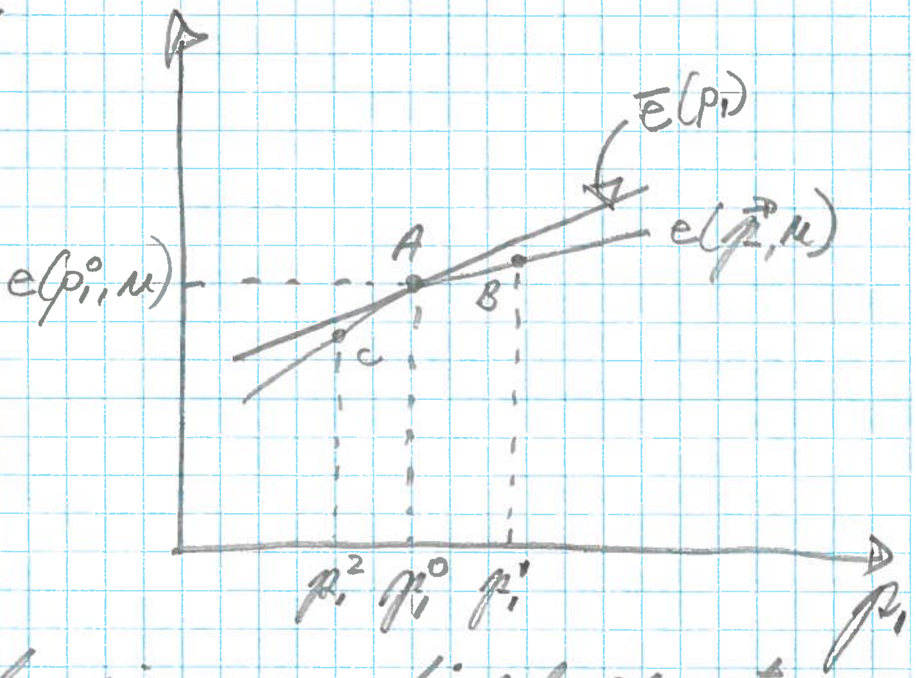
$$\leq t\vec{p}^{\,1}\vec{x}(\vec{p}^{\,t};u)+(1-t)\vec{p}^{\,2}\vec{x}(\vec{p}^{\,t};u)$$

$$\Rightarrow t\,e(\vec{p}^{\,1};u)+(1-t)\,e(\vec{p}^{\,2};u)\leq \vec{p}^{\,t}\vec{x}(\vec{p}^{\,t};u)=e(\vec{p}^{\,t};u)$$

QED.

c) No. Convexity of preferences play no role in order to explain the concavity of the expenditure function. The only property that was invoked is that of expenditure minimization in expressions [A] & [B] above. The following graph provides an intuitive explanation.

Assume that only p_1 varies:



Assume that all prices are fixed except p_1. The initial prices are $\vec{p}^{\,0}$, with associated expenditure $e(\vec{p}^{\,0},u)$ at point A.

We have:

$$e(\vec{p}^{\,0}, u) = p_1^0 x_1(\vec{p}^{\,0}, u) + p_2^0 x_2(\vec{p}^{\,0}, u) + \ldots + p_m^0 x_m(\vec{p}^{\,0}, u)$$

Assume that the consumer does not respond to changes in p_1. Then $\frac{\partial e}{\partial p_1} = x_1(\vec{p}^{\,0}, u)$, i.e., the expenditure function would be linear in p_1, as depicted by line $\bar{e}(p_1)$

But the consumer will respond to a change in p_1 in order to attain utility u at a lower cost than $\bar{e}(p_1)$. Hence, we expect to have $e(\vec{p}^{\,1}, u) < \bar{e}(p_1)$, where $\vec{p}^{\,1} = (p_1^1, p_2^0, \ldots, p_m^0)$, as in point B. And so on for $\vec{p}^{\,2} = (p_1^2, p_2^0, \ldots, p_m^0)$ at point C, ...

#2) Show that $u(\vec{x})$ is quasi-concave iff $\succsim$ is convex.

(see solution to assigned problem 1.23.)

#3) $u(x_1, x_2) = (x_1 - \alpha_1)^{1-\theta} (x_2 - \alpha_2)^{\theta}$, $\theta \in (0,1)$

a) max problem:

$$\max_{x_1, x_2} u(x_1, x_2) \text{ s.t. } p_1 x_1 + p_2 x_2 = y$$

(NB: We impose a binding budget constraint since utility is strictly increasing in x_1 & x_2.)

b) $\mathcal{L} = u(x_1, x_2) + \lambda(y - p_1 x_1 - p_2 x_2)$

$\mathcal{L}_{x_1} = 0 \Rightarrow \frac{\partial u}{\partial x_1} - \lambda p_1 = 0$

$$\Rightarrow (1-\theta)\left(\frac{x_2 - \alpha_2}{x_1 - \alpha_1}\right)^{\theta} = \lambda p_1 \qquad [A]$$

$\mathcal{L}_{x_2} = 0 \Rightarrow \frac{\partial u}{\partial x_2} - \lambda p_2 = 0 \Rightarrow \theta\left(\frac{x_1 - \alpha_1}{x_2 - \alpha_2}\right)^{1-\theta} = \lambda p_2 \qquad [B]$

$\mathcal{L}_{\lambda} = 0 \Rightarrow y = p_1 x_1 + p_2 x_2$

c) In order to derive the indirect utility fctn $v(\vec{p}, y)$, we begin by deriving the demand functions $x_1(\vec{p}, y)$ and $x_2(\vec{p}, y)$. These demands will then be inserted back into the direct utility fctn.

$[A]$ & $[B] \Rightarrow \dfrac{(1-\theta)\left(\frac{x_2-\alpha_2}{x_1-\alpha_1}\right)^{\theta}}{\theta\left(\frac{x_1-\alpha_1}{x_2-\alpha_2}\right)^{1-\theta}} = \dfrac{p_1}{p_2}$

$$\Rightarrow \frac{p_2x_2 - p_2\alpha_2}{p_1x_1 - p_1\alpha_1} = \frac{\theta}{1-\theta}$$

Since $p_2x_2 = y - p_1x_1$, we get, after rearranging:

$$x_1 = \frac{(1-\theta)(y - p_2\alpha_2) + \theta p_1\alpha_1}{p_1} \qquad [D1]$$

And similarly,

$$x_2 = \frac{\theta(y - p_1\alpha_1) + (1-\theta)p_2\alpha_2}{p_2} \qquad [D2]$$

Those are the (ordinary) demand fctns. Substituting back into $u(x_1, x_2)$, we get, after rearranging:

$$v(\vec{p}, y) = \frac{(1-\theta)^{1-\theta}\theta^{\theta}(y - p_2\alpha_2 - p_1\alpha_1)}{p_1^{1-\theta}p_2^{\theta}}$$

d) The ordinary demd fctns were derived above. See [D1] & [D2].

e) In order to derive the consumer's demand for manufactured goods using the indirect utility function, the easiest is to use Roy's identity, i.e.,

$$x_2(\vec{p}, y) = -\frac{\partial v / \partial p_2}{\partial v / \partial y}. \quad \text{THE END.}$$

(THIS NEXT PART WAS NOT REQUIRED.)

$$\frac{\partial v}{\partial p_2} = \frac{(1-\theta)^{1-\theta}\theta^{\theta}(-\alpha_2)}{p_1^{1-\theta} p_2^{\theta}} - \frac{(1-\theta)^{1-\theta}\theta^{\theta}(y - p_2\alpha_2 - p_1\alpha_1)}{(p_1^{1-\theta} p_2^{\theta})^2} \cdot \theta p_1^{1-\theta} p_2^{\theta-1}$$

$$\frac{\partial v}{\partial y} = \frac{(1-\theta)^{1-\theta}\theta^{\theta}}{p_1^{1-\theta} p_2^{\theta}}$$

$$\Rightarrow x_2(\vec{p}, y) = -\left[(-\alpha_2) - \frac{(y - p_2\alpha_2 - p_1\alpha_1)\theta p_2^{\theta-1}}{p_2^{2\theta}} \cdot p_2^{\theta}\right]$$

$$= \frac{\theta(y - p_2\alpha_2 - p_1\alpha_1) + p_2\alpha_2}{p_2}$$

This effectively corresponds to the demand function [D2].