SOLUTION MICRO IV
MID-TERM 1, SUMMER 2015

#2 A game of war

The question is to find one B.N.E.

Since the armies are playing a symmetrical game, it is safe to assume that there must exist a B.N.E. where each type is using the same strategy. That reduces the pure strategy BNE possibilities to only four:

S	W	
A	A	→ {Both strong and weak armies attack.
A	NA	→ {Strong armies attack, Weak armies not attack
NA	A	⋮
NA	NA	

Let us see if pure strategies $(S,W)=(A,A)$ can be a N.E. To this end, let us list the gains in all four equilibria:

S	W	$E(\Pi_S)$	$E(\Pi_W)$
A	A	$\frac{1}{2}(M-s)+\frac{1}{2}(-s)=\frac{M}{2}-s$	$\frac{1}{2}(-m)+\frac{1}{2}(-m)=-m$
A	NA	$\frac{1}{2}(M)+\frac{1}{2}(-s)=\frac{M-s}{2}$	0
NA	A	0	$\frac{1}{2}(M)+\frac{1}{2}(-m)=\frac{M-m}{2}$
NA	NA	0	0

In order for (A,A) to be a BNE, none of the army type wishes to deviate given the choice of the other. In the case of a WEAK army type, it can always do better than an expected negative gain ($-m$) by "NOT ATTACKING". Hence, a weak army gains by deviating from (A, A).

Let us see now the case of $(S,W)=(A,NA)$:

i) Assuming that $\underline{M>s}$, a strong army does not do better by choosing "NA" since its gain drops to 0.

ii) If the general of a weak army deviates by "attacking" while expecting that a strong opponent attacks and a weak one does not, the expected gain becomes:

$$\frac{1}{2}(-m)+\frac{1}{2}(M)=\frac{M-m}{2}.$$

If we assume that $\underline{M<m}$, then we have a BNE.

It can be similarly verified that (NA, A) and (NA, NA) are not equilibria.

EXERCIZE:

Discuss the following cases:

$$M>m>s$$

$$m>s>M$$