FINAL EXAM
MICRO IV : ECO 6122
APRIL 2015.

1) The core of an exchange economy

a) Definition: The core is the set of all feasible allocations that cannot be "blocked" by any possible coalition. A blocking coalition is a subset of consumers that can do better (strictly for at least one consumer) by breaking away from the whole original group of consumers.

b) Definition: a Walrasian equilibrium is given by a vector of prices $\vec{p}^*$ for which all the markets clear, i.e., the quantities demanded equal the quantities supplied through endowments.

c) Show that $W(e) \subset C(e)$.
Suppose not, i.e. allocation $\alpha(\vec{p}^*)$ is in $W(e)$ but not in $C(e)$.

$\alpha(\vec{p}^*) \notin C(e) \Rightarrow \exists$ allocation y and a coalition S such that

[A]: $$\sum_{i \in S} \vec{y}^i = \sum_{i \in S} \vec{e}^i$$

AND [B]: $u^i(\vec{y}^i) \geq u^i(\vec{x}^i(\vec{p}^*, \vec{p}^* \vec{e}^i))$, $\forall i \in S$
with at least one strict inequality.

[A] $\Rightarrow$ $$\vec{p}^* \sum_{i \in S} \vec{y}^i = \vec{p}^* \sum_{i \in S} \vec{e}^i \qquad [C]$$

[B] $\Rightarrow$ $\vec{p}^* \vec{y}^i \geq \vec{p}^* \alpha^i(\vec{p}^*, \vec{p}^* \vec{e}^i) = \vec{p}^* \vec{e}^i$, $\forall i \in S$,
with at least one strict. (Otherwise consumers would do better by choosing allocation y if they could afford it.)
But this implies that

$$\vec{p}^* \sum_{i \in S} \vec{y}^i > \vec{p}^* \sum_{i \in S} \vec{e}^i$$

which contradicts [C].

d) Since all core allocations are Pareto efficient (PE), the above result means that Walrasian equilibria (WE) are PE. This was also the case with a barter economy (BE). There is one big difference, however, between a BE and a WE. For a BE to yield a PE equilibrium, everyone needs to know "everything" about everyone else and be able to communicate with everyone. In a WE, all that one needs to know is the vector of prices. There is no need to know anything about other consumers' preferences or endowments and no need to exchange more information. For this reason, the WE seems more realistic from a practical point of view than the BE.

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I) See assigned problem no 4.8.

II) See assigned problem no 4.22.

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I) See assigned problem 1.42.

II) Walras' law says that

$$\vec{p}\sum_{i\in I}\vec{x}^i = \vec{p}\sum_{i\in I}\vec{e}^i$$

i.e. the aggregate value of expenditure is equal to the aggregate value of endowments.

PROOF: Since u^i is strictly increasing, we have

$$\sum_{k=1}^{n} p_k\left(x_k^i(\vec{p},\vec{p}\,\vec{e}^i) - e_k^i\right) = 0 \quad , \forall i,$$

i.e. budget constraints are binding.

$$\Rightarrow \sum_{i\in I}\sum_{k=1}^{n} p_k\left(x_k^i - e_k^i\right) = 0$$

$$\Rightarrow \sum_{k=1}^{n}\sum_{i\in I} p_k\left(x_k^i - e_k^i\right) = 0$$

$$\Rightarrow \sum_{k=1}^{n} p_k\left(\sum_{i\in I} x_k^i - \sum_{i\in I} e_k^i\right) = 0$$

$$\Rightarrow \sum_{k=1}^{n} p_k \sum_{i\in I} x_k^i = \sum_{k=1}^{n} p_k \sum_{i\in I} e_k^i$$

$$\Rightarrow \vec{p}\sum_{i\in I}\vec{x}^i = \vec{p}\sum \vec{e}^i \qquad QED.$$

Suppose now that markets clear for $k = \{1, 2, \ldots, n-1\}$, that is,

$$\sum_{i\in I} x_k^i = \sum_{i \in I} e_k^i \,, \quad k = 1, \ldots, n-1 .$$

$$\Rightarrow \; p_k \sum_{i\in I} x_k^i = p_k \sum_{i\in I} e_k^i \,, \quad k = 1, \ldots, n-1$$

$$\Rightarrow \; \sum_{k=1}^{n-1} p_k \sum_{i\in I} x_k^i = \sum_{k=1}^{n-1} p_k \sum_{i\in I} e_k^i \qquad [A]$$

According to Walras' Law, we must have:

$$\sum_{k=1}^{n-1} p_k \sum_{i\in I} x_k^i + p_n \sum_{i\in I} x_n^i = \sum_{k=1}^{n-1} p_k \sum_{i\in I} x_k^i + p_n \sum_{i\in I} e_n^i$$

With [A], this implies

$$p_n \sum_{i\in I} x_n^i = p_n \sum_{i\in I} e_n^i$$

$$\Rightarrow \; \sum_{i\in I} x_n^i = \sum_{i\in I} e_n^i .$$

Hence, the last market clears also.

4

a) The lottery is as follows:

① 50% chance of having income $(1+\beta)m$
② 50% chance of having income $(1-\beta)m$

Expected utility is:

$$EU = 0.5\ln((1+\beta)m) + 0.5\ln((1-\beta)m)$$

The certainty equivalent is given by:

$$\ln(c) = 0.5\ln((1+\beta)m) + 0.5\ln((1-\beta)m)$$

$$\Rightarrow \ln(c) = \ln\left[((1+\beta)m)^{0.5}((1-\beta)m)^{0.5}\right]$$

$$c = m\,((1+\beta)(1-\beta))^{0.5}$$

$$c = m\sqrt{1-\beta^2}$$ This is the certainty equivalent.

b) Scarlet's willingness to pay to avoid the gamble is equal to $m-c$. We thus have:

$$WTP = m - c = m - m\sqrt{1-\beta^2} = m\left(1-\sqrt{1-\beta^2}\right)$$

The WTP to avoid the gamble actually increases with income. The problem is that the amounts of the gamble also increase with income. So it is difficult to say anything definite at absolute risk aversion with respect to income levels.

c) The WTP as a proportion of income is

$$\frac{w-c}{w} = 1-\sqrt{1-B^2} .$$

Hence, Scarlet is WTP a constant proportion of her income in order to avoid a bet that is also a constant proportion of her income. For this reason, we say that she displays <u>constant relative risk aversion</u>.

<u>REMARK:</u> The most important in part b) and c) was to look at how the WTP to avoid gambles is related to income levels.