SOLUTIONS
FINAL EXAM ECO 6122
SUMMER 2015.

#1 Proof of existence of a VNM utility fctn

Take any gamble $g \in G$ and define $u(g)$ as follows: $g \sim (u(g) \circ a_1, (1-u(g)) \circ a_n)$.

By the <u>continuity axiom</u>, such a function $u(g)$ must exist. (See axiom 3 in appendix of questionnaire.)

We must now show that $u(g)$ is unique and represents preferences.

<u>1 Uniqueness:</u>

Suppose $u(g)$ is not unique. Then $\exists\, \beta \neq u(g)$ s.t. $g \sim (\beta \circ a_1, (1-\beta) \circ a_n)$.

By transitivity, this implies:

$(u(g) \circ a_1, (1-u(g)) \circ a_n) \sim (\beta \circ a_1, (1-\beta) \circ a_n)$. [*]

Take $\beta > u(g)$, then by the <u>monotonicity axiom</u> we have:

$(\beta \circ a_1, (1-\beta) \circ a_n) \succ (u(g) \circ a_1, (1-u(g)) \circ a_n)$.

(And conversely if $\beta < u(g)$.)

Hence, this contradicts [*].

So $u(g)$ must be unique. <u>QED</u>

2 $u(g)$ represents preferences:

$\Rightarrow$ Show that $g \succsim g' \Leftrightarrow u(g) \geq u(g')$.

By definition of $u(\cdot)$, we have

$g \sim (u(g) \circ a_1, (1-u(g)) \circ a_n)$

$g' \sim (u(g') \circ a_1, (1-u(g')) \circ a_n)$

Hence, by transitivity,

$g \succsim g' \Leftrightarrow (u(g) \circ a_1, (1-u(g)) \circ a_n)$

$\succsim (u(g') \circ a_1, (1-u(g')) \circ a_n)$

By monotonicity, the R.H.S. $\succsim$ above obtains iff $u(g) \geq u(g')$.

Hence $g \succsim g' \Leftrightarrow u(g) \geq u(g')$. QED.

#2| See my solution to problem 3.23 of textbook.

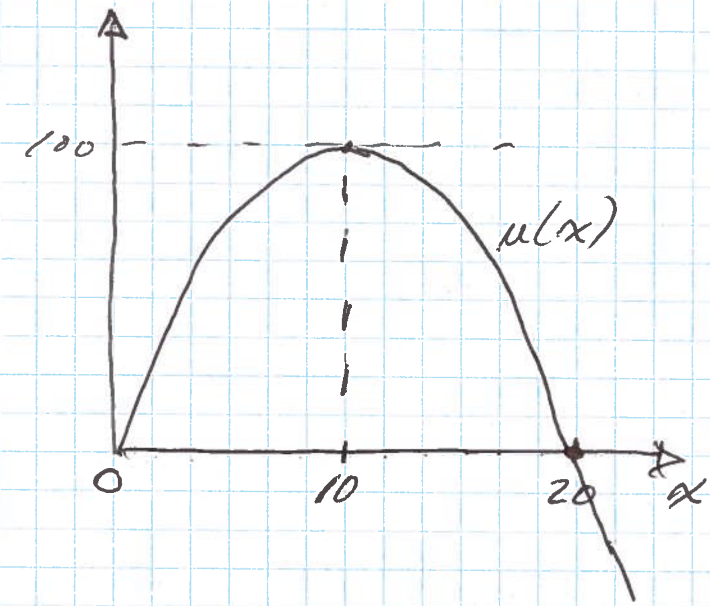
#3| See my other typed solution.

#4 Consumer Choice

$u(x) = 20x - x^2, \ \forall x \geq 0.$

a) $\frac{\partial u}{\partial x} = 20 - 2x \Rightarrow \frac{\partial u}{\partial x} = 0$ at $x = 10.$

$u(x) = 0$ at $x = 0$ and $x = 20$



b) $u = 64$ at $x = 4$ and $x = 16.$

The indifference curve $u = 64$ is composed of the two points $x \in \{4, 16\}.$

Definition of a quasi-concave fctn:

$$f(\vec{x}^*) \geq \min\left[f(\vec{x}^1), f(\vec{x}^2)\right]$$

By <u>graphical inspection</u>, it is straightforward to verify that by connecting any two points x_1 and x_2 (convex combination), all utility levels must be no less than the lowest one $u(x_1)$ or $u(x_2)$.

mathematically, one can simply argue that a concave function must be quasi-concave and that $u(x)$ is concave since $u''(x) < 0$.

c) Lu's problem can be expressed as:

$$\max_x u(x) \text{ s.t. } px \leq m.$$

Obviously, Lu's budget constraint is not binding when $m > p \cdot 10$, but it is binding for $m \leq p \cdot 10$. Formally, one can solve using the Khun-Tucker method (or just use intuitive arguements):

$$\mathcal{L} = 20x - x^2 + \lambda(m - px)$$

$$\Rightarrow \frac{\partial \mathcal{L}}{\partial x} = 20 - 2x - \lambda p = 0$$

and $\lambda(m - px) = 0$, $\lambda \geq 0$, $m - px \geq 0$.

① Binding constraint $\Rightarrow \lambda > 0 \Rightarrow m = px$

$$\Rightarrow x^* = \frac{m}{p}$$

② Non-binding constraint $\Rightarrow \lambda = 0 \Rightarrow m \geq px$

$\Rightarrow 20 - 2x = 0 \Rightarrow x^* = 10 \Rightarrow m \geq 10p$

Hence, the constraint is not binding when $m/p \geq 10$. The demand is thus:

$$x^* = \begin{cases} \frac{m}{p} & \text{for } \frac{m}{p} < 10 \\ 10 & \text{for } \frac{m}{p} \geq 10 \end{cases}$$

d) The indirect utility fctn:

When $\frac{w}{p} < 10$, we have $u(x^*) = 20\frac{w}{p} - \left(\frac{w}{p}\right)^2$

When $\frac{w}{p} \geq 10$, we have $u(x^*) = 20 \cdot 10 - 10^2 = 100$

$$\Rightarrow v(p, w) = \begin{cases} 20\frac{w}{p} - \left(\frac{w}{p}\right)^2 & \text{for } \frac{w}{p} < 10 \\ 100 & \text{for } \frac{w}{p} \geq 10. \end{cases}$$