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**ECO 6122: Microeconomic Theory IV**
Economics Department
University of Ottawa
Mid-term exam
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NB This exam has 2 pages.

**Part A. Producer Theory. (50 points)**

**1. The cost function (20 points)**
**1a) (5 pts)** Define what is a cost function.
**1b) (5 pts)** Suppose that the (one-output, $n$-inputs) production technology exhibits constant returns to scale. Show that the cost function is linear in output.
**1c) (5 pts)** Compare the average and marginal cost functions with constant returns to scale.
**1d) (5 pts)** What can we say about fixed cost with a constant returns to scale technology? Justify.

**2. The technology of production (20 points)**
Consider a single-output, two-input production technology $y = f(x_1, x_2)$.
**2a) (5 pts)** With the help of a graphic, define what is a *convex* input requirement set.
**2b) (5 pts)** Explain **intuitively** what convexity of the input requirement set implies, i.e. why is it a reasonable assumption to make?
**2c) (5 pts)** Can an input requirement set be both convex and non-monotonous? Justify.
**2d) (5 pts)** Can the input requirement set be convex while the production possibilities set is non-convex? Justify.

**3. Duality (10 points)**

A cost function is given by $c(w_1, w_2, y) = [w_1 + w_2]y$.

**3a) (5 pts)** Derive the conditional factor demands.

**3b) (5 pts)** Derive the associated production function.

**Part B. Consumer Theory: True, False or Uncertain. (18 points).**
**Respond to the following 3 questions. Justify your response.**

1. A consumer with a direct utility function $u(x_1, x_2) = \min(x_1, x_2)$ has the following indirect utility function: $v(p, m) = \max\{m / p_1, m / p_2\}$.
2. If $\eta_i = \dfrac{\partial x_i(p,m)}{\partial m}\dfrac{m}{x_i(p,m)}$ and $s_i = \dfrac{px_i(p,m)}{m}$, then it is always true that $\sum_{i=1}^{K} s_i \eta_i = 0$, where $K$ is the number of goods.
3. The substitution matrix is negative semi-definite because the indirect utility function is quasi-convex.

**Part C. Consumer Theory Problems. Respond to the following 2 questions. (32 points)**

4. **(12 points)** Prove that if preferences are strictly convex and prices are all strictly positive, then there is a unique bundle $x(p, m)$ that maximizes utility on the budget set $B(p, m)$.
5. **(20 points)** Suppose a household has the following CES direct utility function, $u(x_1, x_2) = (ax_1^\rho + bx_2^\rho)^{1/\rho}$, where $0 \neq \rho < 1$, and the following budget constraint $p_1x_1 + p_2x_2 \leq m.$.
   a. Find the Marshallian demand functions.
   b. Find the indirect utility function
   c. Show that the indirect utility function is homogenous of degree zero in $p$ and $m$.
   d. Find the expenditure function
   e. Show that the expenditure function is concave in $p$.
   f. Find the Hicksians demand functions