July 21 2015

ECO 6122: Microeconomic Theory IV
Economics Department
University of Ottawa

2nd mid-term exam
Time allotted: 2 hours
Professor: Louis Hotte

NB This questionnaire has 2 pages. Some formulae are provided in the APPENDIX (that you may or may not need).

1. (20 points) The Law of Demand
A decrease in the own price of a normal good will cause quantity demanded to increase. If an own price decrease causes a decrease in quantity demanded, the good must be inferior.

a) (10) Prove each statement in the Law of Demand.

b) (10) Prove that the converse of each statement in the Law of Demand is NOT true.

2. (15 points) The indirect utility function

a) (5) Define the indirect utility function in *both* words and formally.

b) (5) Show that the indirect utility function is homogeneous of degree zero in $(\mathbf{p}, y)$.

c) (5) Demonstrate Roy's identity.

3. (15 points) Consumer demand
Consider the indirect utility function given by

$$v(p_1, p_2, y) = \frac{y}{p_1 + p_2}$$

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a) (5) What are the (ordinary) demand functions?

b) (5) What is the expenditure function?

c) (5) What is the direct utility function?

APPENDIX: Some axioms, definitions and properties

$$\frac{\partial x_i}{\partial p_j} = \frac{\partial x_i^h}{\partial p_j} - x_j \frac{\partial x_i}{\partial y}$$

$$x_i = -\frac{\partial v/\partial p_i}{\partial v/\partial y}$$

$$p_i(\mathbf{x}) = \frac{\partial u/\partial x_i}{\sum_{j=1}^{n} x_j (\partial u/\partial x_j)}$$

Axioms of consumer choice:

1. Completeness. Either $\mathbf{x}^1 \succsim \mathbf{x}^2$ or $\mathbf{x}^2 \succsim \mathbf{x}^1$.
2. Transitivity. If $\mathbf{x}^1 \succsim \mathbf{x}^2$ and $\mathbf{x}^2 \succsim \mathbf{x}^3$, then $\mathbf{x}^1 \succsim \mathbf{x}^3$.
3. Continuity. $\forall \mathbf{x}$ the sets $\succsim (\mathbf{x})$ and $\precsim (\mathbf{x})$ are closed.
4. Strict monotonicity. $\forall \mathbf{x}^0, \mathbf{x}^1$, if $\mathbf{x}^0 \geq \mathbf{x}^1$ then $\mathbf{x}^0 \succsim \mathbf{x}^1$, while if $\mathbf{x}^0 \gg \mathbf{x}^1$ then $\mathbf{x}^0 \succ \mathbf{x}^1$.
5. Convexity. If $\mathbf{x}^0 \succsim \mathbf{x}^1$, then $t\mathbf{x}^1 + (1-t)\mathbf{x}^0 \succsim \mathbf{x}^0, \forall t \in [0,1]$.
6. Strict convexity. If $\mathbf{x}^1 \neq \mathbf{x}^0$ and $\mathbf{x}^1 \succsim \mathbf{x}^0$, then $t\mathbf{x}^1 + (1-t)\mathbf{x}^0 \succsim \mathbf{x}^0, \forall t \in (0,1)$.

Axioms of choice under uncertainty:

1. Completeness. Either $g \succsim g'$ or $g' \succsim g$.
2. Transitivity. If $g \succsim g'$ and $g' \succsim g''$, then $g \succsim g''$.
3. Continuity. $\exists \alpha \in [0,1]$ s.t. $g \sim (\alpha \oplus a_1, (1 - \alpha \oplus a_n))$.
4. Monotonicity. $\forall \alpha, \beta \in [0,1]$, $(\alpha \oplus a_1, (1 - \alpha \oplus a_n)) \succsim (\beta \oplus a_1, (1 - \beta \oplus a_n))$ iff $\alpha \geq \beta$.
5. Substitution. If $g = (p_1 \oplus g_1, ..., p_k \oplus g_k)$ and $h = (p_1 \oplus h_1, ..., p_k \oplus h_k)$ and $h_i \sim g_i, \forall i$, then $h \sim h$.
6. Reduction to simple gambles. ...

Arrow-Pratt measure of risk-aversion:

$$R_a(w) \equiv \frac{-u''(w)}{u'(w)}.$$

Some definitions:

1. Quasiconcave function. $f : D \to R$ is quasiconcave iff, for all $\mathbf{x}^1$ and $\mathbf{x}^2$ in D, $f(\mathbf{x}^t) \geq \min[f(\mathbf{x}^1), f(\mathbf{x}^2)]$ for all $t \in [0,1]$.
2. Concave function. $f : D \to R$ is a concave function if for all $\mathbf{x}^1$ and $\mathbf{x}^2$ in D, $f(\mathbf{x}^t) \geq tf(\mathbf{x}^1) + (1-t)f(\mathbf{x}^2)$ for all $t \in [0,1]$.
3. Convex set. $S \in \Re^n$ is a convex set if for all $\mathbf{x}^1 \in S$ and $\mathbf{x}^2 \in S$, we have $t\mathbf{x}^1 + (1-t)\mathbf{x}^2 \in S$ for all $t \in [0,1]$.