*March 11 2015*

**ECO 6122: Microeconomic Theory IV**
Economics Department
University of Ottawa

2nd mid-term exam
Time allotted: 2 hours
Professor: Louis Hotte

*NB This questionnaire has 3 pages. Some formulae are provided in the APPENDIX (that you may or may not need).*

**1. (15 points) The profit function**

a) (5) Define the profit function in *both* words and formally.

b) (5) Demonstrate (formally) that the profit function is convex in input and output prices.

c) (5) Suppose that the output price fluctuates in such a way that 50% of the times, it equals $0.5P$, and 50% of the times, it equals $1.5P$. Those fluctuations are perfectly predictable. Show that for the producer, this is preferable to a price that is perfectly stable at the mean $P$. (Hint: Use the convexity property.)

**2. (7 points) Integrability Theorem**
Suppose that there are three goods and that a consumer's demand behaviour is summarized by the functions

$$x_i(p_1, p_2, p_3, y) = \frac{\alpha_i y}{p_i}, \quad i = 1, 2, 3,$$

where $\alpha_i > 0$ and $\alpha_1 + \alpha_2 + \alpha_3 = 1$.

a) (5) Suppose that you want to verify that the associated vector of demands $\mathbf{x}(\mathbf{p}, y)$ is utility generated. Describe briefly how you would go about that. (NB I don't expect you to go into the mathematical details here. Just describe the procedure.)

b) (2) What do we mean by "utility generated"?

**3. (15 points) Income and substitution effects**

a) (5) Suppose that there are only two goods to consume, goods 1 and 2. With the help of a two-part graphical representation, explain how the total effect of a decrease in the price of good 1 can be decomposed into a substitution effect and an income effect.

b) (5) Suppose now that there are $n$ goods to consume, $i \in \{1, 2, ..., n\}$, and let $\mathbf{x}(\mathbf{p}, y)$ be the consumer's Marshallian demand system. Define the Slutsky matrix $\mathbf{s}(\mathbf{p}, y)$ of price and income responses.

c) (5) Show that $\mathbf{s}(\mathbf{p}, y)$ must be symmetric and negative semidefinite.

**4. (10 points) Fixed factors**

Prove that short-run supply and short-run variable input demands are homogeneous of degree zero in $p$ and $\mathbf{w}$.

# APPENDIX: Some axioms, definitions and properties

$$\frac{\partial x_i}{\partial p_j} = \frac{\partial x_i^h}{\partial p_j} - x_j \frac{\partial x_i}{\partial y}$$

$$x_i = -\frac{\partial v/\partial p_i}{\partial v/\partial y}$$

$$p_i(\mathbf{x}) = \frac{\partial u/\partial x_i}{\sum_{j=1}^n x_j(\partial u/\partial x_j)}$$

**Axioms of consumer choice:**

1. Completeness. Either $\mathbf{x}^1 \succsim \mathbf{x}^2$ or $\mathbf{x}^2 \succsim \mathbf{x}^1$.
2. Transitivity. If $\mathbf{x}^1 \succsim \mathbf{x}^2$ and $\mathbf{x}^2 \succsim \mathbf{x}^3$, then $\mathbf{x}^1 \succsim \mathbf{x}^3$.
3. Continuity. $\forall \mathbf{x}$ the sets $\succsim(\mathbf{x})$ and $\precsim(\mathbf{x})$ are closed.
4. Strict monotonicity. $\forall \mathbf{x}^0, \mathbf{x}^1$, if $\mathbf{x}^0 \geq \mathbf{x}^1$ then $\mathbf{x}^0 \succsim \mathbf{x}^1$, while if $\mathbf{x}^0 \gg \mathbf{x}^1$ then $\mathbf{x}^0 \succ \mathbf{x}^1$.
5. Convexity. If $\mathbf{x}^0 \succsim \mathbf{x}^1$, then $t\mathbf{x}^1 + (1-t)\mathbf{x}^0 \succsim \mathbf{x}^0, \forall t \in [0,1]$.
6. Strict convexity. If $\mathbf{x}^1 \neq \mathbf{x}^0$ and $\mathbf{x}^1 \succsim \mathbf{x}^0$, then $t\mathbf{x}^1 + (1-t)\mathbf{x}^0 \succsim \mathbf{x}^0, \forall t \in (0,1)$.

**Axioms of choice under uncertainty:**

1. Completeness. Either $g \succsim g'$ or $g' \succsim g$.
2. Transitivity. If $g \succsim g'$ and $g' \succsim g''$, then $g \succsim g''$.
3. Continuity. $\exists \alpha \in [0,1]$ s.t. $g \sim (\alpha \oplus a_1, (1-\alpha \oplus a_n))$.
4. Monotonicity. $\forall \alpha, \beta \in [0,1]$, $(\alpha \oplus a_1, (1-\alpha \oplus a_n)) \succsim (\beta \oplus a_1, (1-\beta \oplus a_n))$ iff $\alpha \geq \beta$.
5. Substitution. If $g = (p_1 \oplus g_1, ..., p_k \oplus g_k)$ and $h = (p_1 \oplus h_1, ..., p_k \oplus h_k)$ and $h_i \sim g_i, \forall i$, then $h \sim h$.
6. Reduction to simple gambles. ...

**Arrow-Pratt measure of risk-aversion:**

$$R_a(w) \equiv \frac{-u''(w)}{u'(w)}.$$

**Some definitions:**

1. Quasiconcave function. $f : D \to R$ is quasiconcave iff, for all $\mathbf{x}^1$ and $\mathbf{x}^2$ in $D$, $f(\mathbf{x}^t) \geq \min[f(\mathbf{x}^1), f(\mathbf{x}^2)]$ for all $t \in [0,1]$.
2. Concave function. $f : D \to R$ is a concave function if for all $\mathbf{x}^1$ and $\mathbf{x}^2$ in $D$, $f(\mathbf{x}^t) \geq tf(\mathbf{x}^1) + (1-t)f(\mathbf{x}^2)$ for all $t \in [0,1]$.
3. Convex set. $S \in \Re^n$ is a convex set if for all $\mathbf{x}^1 \in S$ and $\mathbf{x}^2 \in S$, we have $t\mathbf{x}^1 + (1-t)\mathbf{x}^2 \in S$ for all $t \in [0,1]$.