July 7th 2015

ECO 6122: Microeconomic Theory IV
Economics Department
University of Ottawa

First mid-term exam
Time allotted: 2 hours
Professor: Louis Hotte

NB This questionnaire has 1 page.

1. (50 points) A Tragedy of the Commons

Suppose that two goat herders have access to a pasture. The only input to the pasture takes the form of the number of goats sent grazing. Let X denote the total number of goats. Then $y = f(X)$ is the total output produced in kilograms of meat. To simplify, assume that $f(X) = aX - bX^2$.

Each herder must independently choose the number of goats to send grazing. Let $x_i \in [0, \infty]$ denote that number chosen by herder i, $i \in \{1, 2\}$. We have $X = x_1 + x_2$. Each herder incurs a constant costs c per goat sent grazing and goat meat fetches a constant unit price p per kg. The herders are assumed identical in that each goat input is equally productive at producing meat. Consequently, if $\phi(X)$ denotes the average product of a goat, i.e., $\phi(X) \equiv f(X)/X$, then the total output received by herder i is given by $x_i\phi(X)$. (Remember that $X = x_1 + x_2$ and that due to decreasing returns, we have $\phi'(X) < 0$.)

Herder i's payoff is thus given by $\pi_i = x_i p\phi(x_1 + x_2) - cx_i$, $i \in \{1, 2\}$.

a) Derive each herder's reaction function and illustrate on a graph. Interpret. (Warning: Begin by stating properly the problem of a herder.)

b) Find the Nash equilibrium number of goats chosen by each herder.

c) Find the optimal number of goats X^*, i.e., the value of X that maximizes total profits. Compare this result with the Nash equilibrium total number of goats and comment.

2. (50 points) A Bayesian Game of War

Two opposed armies are poised to seize an island. Each army's general can choose either "ATTACK" or "NOT ATTACK". In addition, each army is either "STRONG" or "WEAK" with equal probability (the draws for each army are independent), and an army's type is known only to its general. Payoffs are as follows: The island is worth M if captured. An army can capture the island either by attacking when its opponent does not or by attacking when its rival does if it is strong and its rival is weak. If two armies of equal strength both attack, neither captures the island. An army has a "cost" of fighting, which is s if it is strong and w if it is weak, where $s < M < w$. There is no cost of attacking if its rival does not.

Identify a Bayesian Nash equilibrium for this game. Make sure to clearly explain all the steps and interpret the results. (Hint: Give the symmetrical nature of the game between types, assume a symmetrical equilibrium where each army plays the same strategy depending on its type.)