February 4th 2015

ECO 6122: Microeconomic Theory IV
Economics Department
University of Ottawa

First mid-term exam
Time allotted: 2 hours
Professor: Louis Hotte

NB This questionnaire has 2 pages. Some formulae are provided in the APPENDIX (that you may or may not need).

1. (20 points) The expenditure function

a) (5) Define the expenditure function in *both* words and formally.

b) (7) Demonstrate (formally) that the expenditure function is concave in prices.

c) (8) Does convexity of preferences play a role in explaining the price-concavity of the expenditure function? Provide an intuitive explanation with the help of a simple graphic.

2. (20 points) A consumption bundle is represented by a vector $\mathbf{x} \in \Re^n_+$. We represent a consumer's preferences by a binary relation $\succsim$, i.e., if $\mathbf{x}^1 \succsim \mathbf{x}^2$, we say that $\mathbf{x}^1$ is at least as good as $\mathbf{x}^2$. Assume that $\succsim$ can be represented by a real-valued function $u : \Re^n_+ \to \Re$, i.e., for all $\mathbf{x}^0$, $\mathbf{x}^1 \in \Re^n_+, u(\mathbf{x}^0) \geq u(\mathbf{x}^1) \Leftrightarrow \mathbf{x}^0 \succsim \mathbf{x}^1$. $u : \Re^n_+ \to \Re$.

Show that $u(\mathbf{x})$ is quasiconcave if and only if $\succsim$ is convex. (Hint: Proceed by contradiction.)

3. (30 points) Suppose that a consumer's welfare depends on the quantities of agricultural goods x_1 and manufactured goods x_2 that she consumes. Suppose more precisely that her utility level can be represented by the following utility function:

$$u(x_1, x_2) = (x_1 - \alpha_1)^{1-\theta}(x_2 - \alpha_2)^{\theta},$$

where α_i are positive parameter values and $\theta \in (0, 1)$. The respective prices of the goods are p_1 and p_2. The consumer's income is y.

a) (5) Write down the consumer's problem.

b) (5) Express the Langragian function for this problem and give the first-order conditions.

c) (10) Derive the *indirect utility function.* (Explain briefly your steps. If you don't, I can't give much partial marks in case you make algebraic mistakes.)

d) (5) Derive the (ordinary) demand functions.

e) (5) If you were provided with the indirect utility function only, how would proceed to find this consumer's demand for manufactured goods?

APPENDIX: Some axioms, definitions and properties

$$\frac{\partial x_i}{\partial p_j} = \frac{\partial x_i^h}{\partial p_j} - x_j \frac{\partial x_i}{\partial y}$$

$$x_i = -\frac{\partial v/\partial p_i}{\partial v/\partial y}$$

$$p_i(\mathbf{x}) = \frac{\partial u/\partial x_i}{\sum_{j=1}^n x_j(\partial u/\partial x_j)}$$

Axioms of consumer choice:

1. Completeness. Either $\mathbf{x}^1 \succsim \mathbf{x}^2$ or $\mathbf{x}^2 \succsim \mathbf{x}^1$.
2. Transitivity. If $\mathbf{x}^1 \succsim \mathbf{x}^2$ and $\mathbf{x}^2 \succsim \mathbf{x}^3$, then $\mathbf{x}^1 \succsim \mathbf{x}^3$.
3. Continuity. $\forall \mathbf{x}$ the sets $\succsim (\mathbf{x})$ and $\precsim (\mathbf{x})$ are closed.
4. Strict monotonicity. $\forall \mathbf{x}^0, \mathbf{x}^1$, if $\mathbf{x}^0 \geq \mathbf{x}^1$ then $\mathbf{x}^0 \succsim \mathbf{x}^1$, while if $\mathbf{x}^0 \gg \mathbf{x}^1$ then $\mathbf{x}^0 \succ \mathbf{x}^1$.
5. Convexity. If $\mathbf{x}^0 \succsim \mathbf{x}^1$, then $t\mathbf{x}^1 + (1-t)\mathbf{x}^0 \succsim \mathbf{x}^0, \forall t \in [0,1]$.
6. Strict convexity. If $\mathbf{x}^1 \neq \mathbf{x}^0$ and $\mathbf{x}^1 \succsim \mathbf{x}^0$, then $t\mathbf{x}^1 + (1-t)\mathbf{x}^0 \succsim \mathbf{x}^0, \forall t \in (0,1)$.

Axioms of choice under uncertainty:

1. Completeness. Either $g \succsim g'$ or $g' \succsim g$.
2. Transitivity. If $g \succsim g'$ and $g' \succsim g''$, then $g \succsim g''$.
3. Continuity. $\exists \alpha \in [0,1]$ s.t. $g \sim (\alpha \oplus a_1, (1-\alpha \oplus a_n))$.
4. Monotonicity. $\forall \alpha, \beta \in [0,1]$, $(\alpha \oplus a_1, (1-\alpha \oplus a_n)) \succsim (\beta \oplus a_1, (1-\beta \oplus a_n))$ iff $\alpha \geq \beta$.
5. Substitution. If $g = (p_1 \oplus g_1, ..., p_k \oplus g_k)$ and $h = (p_1 \oplus h_1, ..., p_k \oplus h_k)$ and $h_i \sim g_i, \forall i$, then $h \sim h$.
6. Reduction to simple gambles. ...

Arrow-Pratt measure of risk-aversion:

$$R_a(w) \equiv \frac{-u''(w)}{u'(w)}.$$

Some definitions:

1. Quasiconcave function. $f : D \to R$ is quasiconcave iff, for all $\mathbf{x}^1$ and $\mathbf{x}^2$ in D, $f(\mathbf{x}^t) \geq \min[f(\mathbf{x}^1), f(\mathbf{x}^2)]$ for all $t \in [0,1]$.
2. Concave function. $f : D \to R$ is a concave function if for all $\mathbf{x}^1$ and $\mathbf{x}^2$ in D, $f(\mathbf{x}^t) \geq tf(\mathbf{x}^1) + (1-t)f(\mathbf{x}^2)$ for all $t \in [0,1]$.
3. Convex set. $S \in \Re^n$ is a convex set if for all $\mathbf{x}^1 \in S$ and $\mathbf{x}^2 \in S$, we have $t\mathbf{x}^1 + (1-t)\mathbf{x}^2 \in S$ for all $t \in [0,1]$.